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Usage of Unsaturated Porous Medium for One Dimensional Flow

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ABSTRACT

We have assumed the permeability of the medium as a specific linear function of moisture content and time. Under certain assumptions the corresponding boundary value problem is derived from the physical phenomenon and is represented in terms of the boundary value problem. Such flows are of great practical importance in water resources science.

Many researchers have discussed such flows from different viewpoints; namely, has utilized a finite difference approach, has used a transformation of variable system; has discussed a variation approach; has obtained a transformation and similarity has obtained an approximate solution by the strategy for singular perturbation procedure. We have, under certain assumptions derived a general solution of such flow using one parameter aggregate theoretic strategy. The partial differential equation is transformed into an ordinary differential equation using one parameter gather theory of similarity analysis. Finally a general solution of corresponding ordinary differential equation is obtained. *The*

current paper highlights the usage of unsaturated porous medium for one dimensional flow.

KEYWORDS:

Dimensional, Flow, Unsaturated, Porous

INTRODUCTION

The motion of water flow through unsaturated porous media is governed by the continuity equation:

$$\frac{\partial(\rho_s .u)}{\partial t} = \nabla . M \quad (1)$$

Where i the bulk density of medium on dry weight basis, u is the moisture content at any depth z on a dry weight basis and M is the mass flux of moisture. From Darcy's law for the motion of water in a porous medium, we have

$$V = - k\nabla \phi \quad (2)$$

Where V is the volume flux of moisture, k is the coefficient of aqueous conductivity and ø is the gradient of the whole moisture potential.

Combining equations (1) and (2), we get

$$\frac{\partial(\rho_s .u)}{\partial t} = \nabla(\rho\, k \nabla \text{ø}) \quad (3)$$

Where is the flux density?

Since the flow takes place only in vertical direction, equation (3) reduces to

$$\rho_s \frac{\partial u}{\partial t} = \frac{\partial}{\partial z}(\rho\, k \frac{\partial \Psi}{\partial z}) + \frac{\partial}{\partial z}\,(\rho\, k\, g) \quad (4)$$

Where Ψ is pressure (capillary) potential, g is the gravitational constant and ø = Ψ– g z

The positive direction of z-axis is the same as that of the gravity. Considering u and Ψ to be connected by a single valued function, (4) can be written as

$$\frac{\partial u}{\partial t} = \frac{\rho_k}{\rho_s}\frac{\partial^2 \Psi}{\partial^2 z} + \frac{\rho}{\rho s}\,\{\frac{\partial \Psi}{\partial z} + g\}\,\frac{\partial k}{\partial z}$$

$$= \frac{\rho}{\rho_s}\frac{\partial k}{\partial z}\frac{\partial \Psi}{\partial z} + \frac{\rho_k}{\rho_s}\frac{\partial^2 \Psi}{\partial^2 z} + \frac{\rho_g}{\rho_s}\frac{\partial k}{\partial z}$$

$$= \frac{\partial}{\partial z}\,\{\frac{\rho_k}{\rho_s}\frac{\partial \Psi}{\partial z}\} + \frac{\rho_g}{\rho_s}\frac{\partial k}{\partial z}$$

$$\therefore \quad \frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\{D\,\frac{\partial u}{\partial z}\} + \frac{\rho_g}{\rho_s}\frac{\partial k}{\partial z} \quad (5)$$

$$\text{where } D = \frac{\rho_k}{\rho_s}\frac{\partial \Psi}{\partial u}$$

Which is called the diffusivity coefficient?

It is assumed that the diffusivity coefficient D is equivalent to the average value over the entire range of the moisture content. Furthermore, the permeability k of the medium is considered to vary straightforwardly with the moisture content u and inversely as the square base of time t.

Therefore the equation (5) becomes

$$\frac{\partial u}{\partial t} = D_a \frac{\partial^2 u}{\partial^2 z} + \frac{\rho_g}{\rho_s}\frac{k_0}{\sqrt{t}}\frac{\partial u}{\partial z} \quad (6)$$

$$\text{where } k = \frac{k_0}{\sqrt{t}} u \; ; \quad (k_0 = 0.232) \quad (7)$$

For definiteness of the physical problem, we consider here that the recharge takes place over a large basis of such geological configuration that the sides are limited by unbending boundaries while the bottom is confined by a thick layer of water table. In these circumstances, water will flow vertically downwards under the following boundary conditions:

$$u(0,t) = u_0\ ;\ u(L,t) = u_L \text{ (where } u_L \neq 1) \quad (8)$$

Thus, we have the following boundary value problem for the flow mentioned above

$$\frac{\partial u}{\partial t} = D_a \frac{\partial^2 u}{\partial z^2} + \frac{\rho_g}{\rho_s}\frac{k_0}{\sqrt{t}}\frac{\partial u}{\partial z} \quad \text{(From 8)}$$

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The partial differential equation (6) is transformed into ordinary differential equation by using a similarity variable. The later equation is solved using dimensionless variables and one parameter assembles theory of similarity analysis. Finally analytical solution is obtained.

Considering, (6) and (8)

$$\text{i.e.} \quad \frac{\partial u}{\partial t} = D_a \frac{\partial^2 u}{\partial^2 z} + \frac{\rho_g}{\rho_s}\frac{k_0}{\sqrt{t}}\frac{\partial u}{\partial z} \quad \text{and}$$

$$u(0,t) = u_0\ ;\ u(L,t) = u_L \text{ (where } u_L \neq 1)$$

$$\text{where } \frac{\rho_g}{\rho_s}\frac{k_0}{\sqrt{t}} = \frac{u}{L}$$

We get,

$$\frac{\partial u}{\partial t} = D_a \frac{\partial^2 u}{\partial z^2} + \frac{u}{L}\frac{\partial u}{\partial z} \quad (9)$$

Using dimensionless variables Z and T as

$$Z = \frac{z}{L}, \quad T = \frac{t}{L^2}$$

$$\Longrightarrow \frac{\partial Z}{\partial z} = \frac{1}{L}, \quad \frac{\partial T}{\partial t} = \frac{1}{L^2}$$

Now

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial T} \cdot \frac{\partial T}{\partial t} = \frac{1}{L^2} \cdot \frac{\partial u}{\partial T}$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial Z} \cdot \frac{\partial Z}{\partial z} = \frac{1}{L} \cdot \frac{\partial u}{\partial Z}$$

$$\therefore \frac{\partial^2 u}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{1}{L} \cdot \frac{\partial u}{\partial Z}\right) = \frac{1}{L} \cdot \frac{\partial}{\partial Z}\left(\frac{\partial u}{\partial Z}\right) \frac{\partial Z}{\partial z} = \frac{1}{L^2}\frac{\partial^2 u}{\partial Z^2}$$

Equation (1) reduces to

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$$\frac{1}{L^2} \cdot \frac{\partial u}{\partial T} = D_a \frac{1}{L^2} \frac{\partial^2 u}{\partial Z^2} + \frac{u}{L} \cdot \frac{1}{L} \frac{\partial u}{\partial Z}$$

$$\frac{\partial u}{\partial T} = D_a \frac{\partial^2 u}{\partial Z^2} + u \frac{\partial u}{\partial Z} \quad (10)$$

The boundary conditions are transformed to

When $z = 0 \Longrightarrow Z = \frac{z}{L} = 0$

When $z = L \Longrightarrow Z = \frac{L}{L} = 1$

$\therefore u(0,T) = u\ (0, \frac{t}{L^2}) = u_0$

$u(L,\ T) = u\ (L, \frac{t}{L^2}) = u_L$ (11)

$u(L,\ T) = u\ (L, \frac{t}{L^2}) = u_L$ (12)

Thus, the boundary value problem (6) and (7) is transformed into the following boundary value problem

$$\frac{\partial u}{\partial T} = D_a \frac{\partial^2 u}{\partial Z^2} + u \frac{\partial u}{\partial Z}$$

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$$u(0, T) = u_0$$

$$u(L, T) = u_L$$

Let the solution of the problem (2) be given by

$$u(Z, T) = G(Z, T). H(T)$$

\

i.e. $u = GH$ (13)

Now

$$\frac{\partial u}{\partial T} = H\frac{\partial G}{\partial T} + G\frac{\partial H}{\partial T}; \quad \frac{\partial u}{\partial Z} = \frac{\partial}{\partial Z}(GH) = H\frac{\partial G}{\partial Z}$$

$$\frac{\partial^2 u}{\partial Z^2} = \frac{\partial}{\partial Z}(H\frac{\partial G}{\partial Z}) = H\frac{\partial^2 G}{\partial Z^2}$$

The differential equation (2) takes the form

$$H\frac{\partial G}{\partial T} + G\frac{\partial H}{\partial T} = D_a H\frac{\partial^2 G}{\partial Z^2} + GH(H\frac{\partial G}{\partial Z})$$

$$H\frac{\partial G}{\partial T} + G\frac{\partial H}{\partial T} = D_a H\frac{\partial^2 G}{\partial Z^2} + GH^2\frac{\partial G}{\partial Z} \quad (14)$$

Hence, the boundary conditions (3) and (4) are transformed into: (7)

$$\text{When } Z = 0 \Rightarrow \; u(0,T) = G(0,T).H(T) \Rightarrow u_0 = G(0,T)\; H(T)$$
$$\text{When } Z = L \Rightarrow \; u(L,T) = G(L,T).H(T) \Rightarrow u_L = G(L,T)\; H(T) \quad (15)$$

CONCLUSION

The Flow through saturated-unsaturated porous media is extremely important in various natural and industrial based applications. While the Darcy's law with various modifications are used to model the flow through a porous media, the flow through unsaturated porous media is largely based on conservation of mass and modified Darcy's law where non-linear relationship exists between the pressure head and the fluid saturation coupled with fluid density variations.

Conservation of mass and momentum equation are used for pressure variation measurement. Based on pressure variation, properties based on capillary pressure, pressure head variation, saturation variation of water and oil can be calculated to plot/ evaluate relationship to better understanding of unsaturated flow problem deals in porous media. Above approach can be extended to develop a reliable laboratory scale model to investigate the flow related problem characteristic through porous media.

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