Application Of Cusum Chart In Control Of Paper Quality

S.Ramesh¹ and B.A.Vasu²

ABSTRACT

This paper is an attempt to assess if the manufacturing process of paper machine is in statistical control thereby improving the quality of paper being produced in a paper industry at the time of process itself. Quality is the foremost criteria for achieving the business target. Therefore, emphasis was made on controlling the quality of paper at the time of manufacturing process itself, rather than checking the finished lots at a later time. This control on quality will help the industry deduct the small shift in the process parameters and modify the operating characteristics at the time of production itself rather than receiving complaints from customers at a later stage. This paper describes controlling quality at the time of manufacture itself and helps the industry to concentrate on quality at low cost. The researcher has collected primary data at a leading paper industry during October, 2019. Though X-bar and Range charges were primarily used, CUSUM charts were used to sense the minor shifts in manufacturing process, to explore the possibility of adjusting process parameters during manufacture of paper.

KEYWORDS: paper machine, statistical process control, CUSUM control chart

INTRODUCTION

A product’s quality can be evaluated in different dimensions, viz. Performance, Reliability, Durability, Serviceability, Aesthetics, Features, Perceived Quality, Conformance to Standards, etc. (Montgomery, 1996). The quality can be understood as appropriateness for use. The modern definition relates quality to variability, i.e., quality is inversely proportional to variability. More variation in the production process may results in wastage. In other words, quality can also be understood as reduction of waste in production process.

OBJECTIVES

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• To assess whether the quality of the product produced at paper mill conforms to specifications.

• To assess whether the manufacturing process is in control.

• To forecast the possible deviations in process in future.

METHODOLOGY

The population selected for the study is Paper came out of a Paper Mill. Primary data was gathered for the research project. Data were collected during the manufacturing process of the finished product. The measurements on Substance, Caliper and Bulk Density of paper were taken for study. Bulk Density is the ratio of Substance to Caliper. These measurements were collected for sample size of 20 at different time periods during the manufacturing process with sample subgroup size of 20.

SQC TOOLS

The control chart technique was used for the study. Control chart is a graphical representation of measurements of a product or item at different points of time. It will reveal any potential problems in the production process. The idea behind control chart the average of measurements of every set of few items being studied in different time periods may not vary too much throughout the period of study. The mean of all measurements will be lying in the middle and the variations will not be far away from it. In statistics, the term “far away” implies more than several standard deviations from mean. The control chart is based on the area property of normal distribution and generally deploys 3-sigma limits on either side of mean thereby leaving little chance for few measurements exceeding the limits, if the process is in control. The control chart will have a middle line (Central line – process mean), upper control limit (UCL- desired standard deviation above grand mean) and the lower control limit (LCL- desired standard deviation below grand mean). When the measurements of the item under study fall beyond these upper and lower control limits, the process is regarded to be out of control. A control chart is portrayed in Figure 1.
Figure 1

**Control Chart for $\bar{X}$ (Mean)**

The control chart for $\bar{X}$ uses subgroups of size n each for k consecutive periods of time. To compute control limits, the grand mean of all the subgroup means ($\bar{X}$) and their standard deviation ($\sigma_X$) are calculated.

Control Limits for X-bar Chart (when $\sigma$ is not known)

- **CL** = $\bar{X}$
- **UCL Limit** = $\bar{X} + 3 \frac{\bar{R}}{d_2\sqrt{n}}$
- **LCL** = $\bar{X} - 3 \frac{\bar{R}}{d_2\sqrt{n}}$

where $\bar{X} = \frac{\sum_{i=1}^{k} X_i}{k}$; $\bar{R} = \frac{\sum_{i=1}^{k} R_i}{k}$; $\frac{\bar{R}}{d_2}$ is estimate of $\sigma$.

- $\bar{X}_i$ = sample mean of $i^{th}$ subgroup
- $R_i$ = range of $i^{th}$ subgroup
- k = number of subgroups
- $d_2$ is relationship factor between standard deviation and range, to be obtained from Tables.

After simplifying the calculations by utilizing $A_2$ factor which is equal to $\frac{3}{d_2\sqrt{n}}$, the simplified control limits are:

- **CL** = $\bar{X}$
- **UCL** = $\bar{X} + A_2 \bar{R}$
- **LCL** = $\bar{X} - A_2 \bar{R}$

The values of $A_2$ can be obtained from Statistical Tables for different values of n.

**Control Chart for R (Range)**

The R chart determines as to whether the process variability is in control or varying according to time.

**Control Limits for the R Chart**

- **CL** = $\bar{R}$
- **UCL** = $D_4 \bar{R}$
LCL  =  \(D_3 \bar{R}\)

where  \(\bar{R} = \frac{\sum R_i}{k}\)

\(R_i\) = range of \(i^{th}\) subgroup
\(k\) = number of subgroups

The values of \(D_3\) and \(D_4\) can be obtained from Statistical Tables for different values of \(n\).

**Control Chart for \(\bar{X}\) and S**

Even though the X-bar and R charts are widely used, sometimes it is desirable to calculate standard deviation directly instead of estimating through R, and therefore we will have control charts for \(\bar{X}\) and S, where S is the sample standard deviation. Generally, \(\bar{X}\) and S charts are desirable when the sample is more than 10 or varying.

Control Limits for the S Chart

\[
\text{CL} = \bar{S} \\
\text{UCL} = B_4 \bar{S} \\
\text{LCL} = B_3 \bar{S}
\]

The values of \(B_3\) and \(B_4\) can be obtained from Statistical Tables for different values of \(n\).

**Cumulative Sum Control Chart**

The above Control Charts may not be effective when the shift in the process in very small, and cumulative sum (or cusum) control chart may be a better alternative when small shifts will affect the product quality. It directly plots the cumulative sums of the deviations of the sample measurements from a standard value. There are two types of cusum charts available, tabular and v-mark, out of this, tabular cusum is preferable.

The deviations above from standard value \(\mu_0\) are accumulated with a statistic \(C^+\) and the deviations below the standard value \(\mu_0\) is accumulated with the statistic \(C^-\). These are the one-sided upper and lower cusums, respectively and are calculated as follows:

\[
C^+_i = \max[0, x_i - (\mu_0 + K) + C^+_{i-1}] \\
C^-_i = \max[0, (\mu_0 - K) - x_i + C^-_{i-1}]
\]
where the initial values are $C_0^+ = C_0^- = 0$

K is reference value, and chosen as halfway from standard value $\mu_0$ and the shifted value $\mu_1$. Thus K is half the measure of the swing or

$$K = \frac{|\mu_1 - \mu_0|}{2}$$

$C_i^+$ and $C_i^-$ accumulate deviations from standard value $\mu_0$ that are above K, and will be reset to zero if it is negative. If any of $C_i^+$ or $C_i^-$ go beyond decision interval H, the process will be regarded as out of control. A reasonable value for H is five times the process standard deviation $\sigma$.

### IMPLICATIONS OF STUDY

**Table 1 - Measurements on Paper attributes**

<table>
<thead>
<tr>
<th>Roll No.</th>
<th>Caliper</th>
<th>Substance</th>
<th>Bulk Density = $\frac{\text{Caliper}}{\text{Substance}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>98.65</td>
<td>75.09</td>
<td>1.31</td>
</tr>
<tr>
<td>2</td>
<td>100.00</td>
<td>75.45</td>
<td>1.33</td>
</tr>
<tr>
<td>3</td>
<td>99.10</td>
<td>75.64</td>
<td>1.31</td>
</tr>
<tr>
<td>4</td>
<td>100.50</td>
<td>74.82</td>
<td>1.34</td>
</tr>
<tr>
<td>5</td>
<td>98.00</td>
<td>74.18</td>
<td>1.32</td>
</tr>
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<td>6</td>
<td>98.40</td>
<td>74.45</td>
<td>1.32</td>
</tr>
<tr>
<td>7</td>
<td>97.05</td>
<td>73.55</td>
<td>1.32</td>
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<td>8</td>
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</tr>
<tr>
<td>20</td>
<td>102.30</td>
<td>75.91</td>
<td>1.35</td>
</tr>
</tbody>
</table>

The above data were analyzed through $\bar{X}$ Chart, R and s charts for Caliper and Substance and CUSUM chart was used to analyze the Bulk Density to determine the small shift in the process.
The X-bar charts for both Caliper and Substance show the process not in control, but however, the R and S charts show there is not much variability in values. The measurements for bulk density were analyzed through CUSUM charts because X-bar and R/S charts are incapable of measuring the little shifts in the process.

The chart shows that there is a 1σ shift in the process, even though all the measurements plot within the control limits. Hence it can be seen that CUSUM chart is capable of detecting a small shift.

Assumptions:
1. Process Mean is between 1.32 and 1.35, i.e. 1.335
2. σ is standard deviation of sample in forming cusum (0.005).
3. K is selected half way between process mean and shifted mean (1σ).
4. H is assumed 5 times of standard deviation of sample variable, i.e. 0.025
The upper side cusum $C_{14}^+ > H (=0.025)$, the process is not in control at this point. The process was in control at period 14-1 =13, so the swing is likely to have occurred between periods 13 and 14. The operating parameters should be checked before taking run after the gap.

CONCLUSION

Although the Range/S charts do not show much variation in the process, the CUSUM charts clearly indicates there is a shift in the process. Therefore, it is imperative in the paper industry, to use CUSUM chart, even when R/S charts show the process is under control. If proper tool is not selected for checking quality, it may lead to other problems in quality. Hence, using the traditional control charts like mean, range and standard deviation may sometimes lead to unfair conclusions. The CUSUM chart may be a suitable alternative.

REFERENCES


