

The Vogel's Approximation Algorithm And Solutions

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Abstract

The Vogel's approximation algorithm and solution for transportation problem is considered in this paper. The objective in a transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost. Whenever there is a physical movement of goods from the point of manufacturer to the final consumers through a variety of channels of distribution (wholesalers, retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase profit on sales. The data obtained were modelled as a special Linear Programming problem. The MATLAB software is used to generate tableau for an optimal solution and the transportation problem was formulated and initial feasible solution were obtained using VAM, the solution obtained was tested for optimality using MODI. The model is useful for making strategic decisions involved in selecting optimum transportation routes so as to allocate the production of various plants to several warehouses or distribution centers.

Keywords: Optimization, Constraint, Transportation, Optimal test.

1. Introduction: The transportation problem (TP) is an important Linear Programming (LP) model that arises in several contexts and has deservedly received much attention in literature.

The transportation problem is probably the most important special linear programming problem in terms of relative frequency with which it appears in the applications and also in the simplicity of the procedure developed for its solution. The following features of the transportation problem are considered to be most important.

The TP were the earliest class of linear programs discovered to have totally unimodular matrices and integral extreme points resulting in considerable simplification of the simplex method.

The study of the TP laid the foundation for further theoretical and algorithmic development of the minimal cost network flow problems. Transportation problem is famous in operation research for its wide application in real life. This is a special kind of the network optimization problems in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the source and destination, respectively, such that the total cost of transportation is minimized.

The basic transportation problem was originally developed by (Hitchcock 1941). Aashtiani and Mohammadian (2009) indicated that the transportation solution problem can be found with a good success in the improving the service quality of the public transport systems. Ahmed et al (2016) investigated TP with a new approach named allocation table method (ATM) for finding an initial basic feasible solution of transportation problems, efficiency of allocation table method has also been tested by solving several number of cost minimizing transportation problems and it is found that the allocation table method yields comparatively a better result. The BCM obtained the optimal solution or the closest to optimal solution with a minimum computation time. As well as, using BCM will reduce the complexity with a simple and a clear solution manner which is easily used on different area for optimization problems, (Hlayel and Alia 2012).

Minimization of transportation time is a great concern of the transportation problems like the cost minimizing transportation problems. In this writing, a transportation algorithm is developed and applied to obtain an Initial Basic Feasible Solution (IBFS) of transportation problems in minimizing transportation time. The developed method has also been illustrated numerically to test the efficiency of the method where it is observed that the proposed method yields a better result, (Ahmed et al 2015).

2. Model Formulation of Problem:

The transportation problem deals with a situation in which a single product is to be transported from several sources (also call origin, supply or capacity centers) to several sinks (also called destination, demand or requirement centers). In general, let there be m sources S_1, S_2, \dots, S_m having a_i ($i = 1, 2, \dots, m$) units of supplies or capacity respectively to be transported among n destinations, D_1, D_2, \dots, D_n with

b_j ($j = 1, 2, \dots, n$) units of requirement, respectively. Let c_{ij} be the cost of shipping one unit of the commodity from source i to destination j for each route. If x_{ij} represent the number of units shipped per route from source i to destination j then the problem is to determine the transportation schedule so as to minimize the total transportation cost satisfying supply and demand conditions. Mathematically, the problem may be stated as follows:

$$\text{Minimize (total cost) } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \dots\dots\dots (1)$$

Subject to the constraints

$$\sum_{j=1}^n x_{ij} = a_i, \quad i = 1, 2, \dots, m \text{ (Supply constraints)} \dots\dots\dots (2)$$

$$\sum_{i=1}^m x_{ij} = b_j, \quad j = 1, 2, \dots, n \text{ (Demand constraints)} \dots\dots\dots (3)$$

And $x_{ij} \geq 0$ for all i and j

The transportation problem can be described using linear programming mathematical model and usually it appears in a transportation tableau.

The model of a transportation problem can be represented in a concise tabular form with all the relevant parameters.

The transportation tableau (A typical TP is represented in standard matrix form), where supply availability (a_i) at each source is shown in the far right column and the destination requirements (b_j) are shown in the bottom row. Each cell represents one route. The unit shipping cost (C_{ij}) is shown in the upper right corner of the cell, the amount of shipped material is shown in the center of the cell. The transportation tableau implicitly expresses the supply and demand constraints and the shipping cost between each demand and supply point.

VOGEL’S APPROXIMATION METHOD (VAM)

VAM is an improved version of the least-cost method that generally, but not always, produces better starting solutions. VAM is based upon the concept of minimizing opportunity (Or penalty) costs. The opportunity cost for a given supply row or demand column is defined as the difference between the lowest cost and the next lowest cost alternative. This method is preferred over the methods discussed above because it generally yields, an optimum, or close to optimum, starting solutions. Consequently, if we use the initial solution obtained by VAM and proceed to solve for the optimum solution, the amount of time required to arrive at the optimum solution is greatly reduced. The steps involved in determining an initial solution using VAM are as follows: The steps involved in determining an initial solution using VAM are as follows:

- Step1. Write the given transportation problem in tabular form (if not given).
- Step2. Compute the difference between the minimum cost and the next minimum cost corresponding to each row and each column which is known as penalty cost.
- Step3. Choose the maximum difference or highest penalty cost. Suppose it corresponds to the i th row. Choose the cell with minimum cost in the i th row. Again if the maximum corresponds to a column, choose the cell with the minimum cost in this column.
- Step4. Suppose it is the (i, j) th cell. Allocate $\min(a_i, b_j)$ to this cell. If the $\min(a_i, b_j) = a_i$, then the availability of the i th origin is exhausted and demand at the j th destination remains as $b_j - a_i$ and the i th row is deleted from the table. Again if $\min(a_i, b_j) = b_j$, then demand at the j th destination is fulfilled and the availability at the i th origin remains to be $a_i - b_j$ and the j th column is deleted from the table.

Step5. Repeat steps 2, 3, and 4 with the remaining table until all origins are exhausted and all demands are fulfilled.

- Method is based on the concept of penalty cost or regret.
- A penalty cost is the difference between the largest and the next largest cell cost in a row (or column).
- In VAM the first step is to develop a penalty cost for each source and destination.
- Penalty cost is calculated by subtracting the minimum cell cost from the next higher cell cost in each row and column.

Vogel's Approximation Method (VAM) Summary of Steps

1. Determine the penalty cost for each row and column.
2. Select the row or column with the highest penalty cost.
3. Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.
4. Repeat steps 1, 2, and 3 until all rim requirements have been met

Let Y_1 and Y_2 = plant site

X_{ij} = the units shipped in trucks from plant i to construction sites j

Using the transportation cost data, the weekly transportation cost in thousands of naira is written as:

Minimize

$$50x_{11} + 85x_{12} + 90x_{13} + 30x_{14} + 50x_{15} + 60x_{16} + 40x_{17} \\ + 100x_{21} + 85x_{22} + 50x_{23} + 70x_{24} + 80x_{25} + 80x_{26} + 90x_{27}$$

Consider capacity constraint

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} \leq 96$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} \leq 48$$

Demand constraint

$$x_{11} + x_{21} = 22$$

$$x_{12} + x_{22} = 28$$

$$x_{13} + x_{23} = 18$$

$$x_{14} + x_{24} = 15$$

$$x_{15} + x_{25} = 18$$

$$x_{16} + x_{26} = 19$$

$$x_{17} + x_{27} = 24$$

$$x_{ij} \geq 0 \quad \text{For all } i \text{ and } j$$

3. Solution of the Problem: Matrix Solution of Problem

F= [50 85 90 30 50 60 40 100 85 50 70 80 80 90];

A=[1 1 1 1 1 1 1 0 0 0 0 0 0 0;0 0 0 0 0 0 0 1 1 1 1 1 1 1;1 0 0 0 0 0 0 1 0 0 0 0 0 0;0 1 0 0 0 0 0 0 1 0 0 0 0 0;0 1 0 0 0 0 0 0 1 0 0 0 0 0 1];

b= [96 48 22 28 18 15 18 19 24];

x=A\b

x =

22.0000
0
18.0000
0
18.0000
0
38.0000
0
28.0000
0
15.0000
0
19.0000
-14.0000

Z = #9100

4. Vogel's Approximation Method Solution

TABLE 1: The VAM Table

	A	B	C	D	E	F	G	CAPACITY
Y ₁	50	85	90	30	50	60	40	2880
Y ₂	100	83	50	70	80	80	90	1440
DEMAND	660	840	540	450	540	570	720	4320

Table 2: The VAM penalty cost

	A	B	C	D	E	F	G	CAPACITY	Diff
Y ₁	50	85	90	30	50	60	40	96	10
Y ₂	100	83	50	70	80	80	90	48	20
DEMAND	22	28	18	15	18	19	24	144	
Difference	50	2	40	40	30	20	50		

Table 3: The Initial VAM Allocation

VAM allocates as much as possible to the minimum cost cell in the row or column the largest penalty cost.

	A	B	C	D	E	F	G	CAPACITY	Diff
Y ₁	50 22	85	90	30	50	60	40 24	74	10
Y ₂	100	83	50	70	80	80	90	48	20
DEMAND	22	28	18	15	18	19	24	144	
Difference	50	2	40	40	30	20	50		

Table 4: The second VAM Allocation

	A	B	C	D	E	F	G	CAPACITY	Diff
Y ₁	50 22	85	90	30 15	50	60	40 24	50	20
Y ₂	100	83	50	70	80	80	90	48	20
DEMAND	22	28	18	15	18	19	24	144	
Difference	50	2	40	40	30	20	50		

Table 5: The third VAM Allocation

	A	B	C	D	E	F	G	CAPACITY	Diff
Y ₁	50 22	85	90	30 15	50	60	40 24	35	10
Y ₂	100	83	50 18	70	80	80	90	48	30
DEMAND	22	28	18	15	18	19	24	144	
Difference	50	2	40	40	30	20	50		

Table 6: The fourth VAM Allocation

	A	B	C	D	E	F	G	CAPACITY	Diff
Y ₁	50 22	85	90	30 15	50	60	40 24	35	10
Y ₂	100	83	50 18	70	80	80	90	30	0
DEMAND	22	28	18	15	18	19	24	144	
Difference	50	2	40	40	30	20	50		

Table 7: The fifth VAM Allocation

	A	B	C	D	E	F	G	CAPACITY	Diff
Y ₁	50 22	85	90	30 15	50 18	60	40 24	17	25
Y ₂	100	83	50 18	70	80	80	90	30	3
DEMAND	22	28	18	15	18	19	24	144	
Difference	50	2	40	40	30	20	50		

Table 8: The sixth VAM Allocation

	A	B	C	D	E	F	G	CAPACITY	diff
Y ₁	50 22	85	90	30 15	50 18	60 17	40 24	17	25
Y ₂	100	83	50 18	70	80	80	90	30	3
DEMAND	22	28	18	15	18	19	24	144	
Difference	50	2	40	40	30	20	50		

Table 9: The seventh VAM Allocation

	A	B	C	D	E	F	G	CAPACITY	Diff
Y ₁	50 22	85	90	30 15	50 18	60 17	40 24	17	25

Y ₂	100	83	50	70	80	80	90	30	3
			18			2			
DEMAND	22	28	18	15	18	2	24	144	
Difference	50	2	40	40	30	20	50		

Table 10: The eight VAM Allocation

	A	B	C	D	E	F	G	CAPACITY	Diff
Y ₁	50	85	90	30	50	60	40	17	25
	22			15	18	17	24		
Y ₂	100	83	50	70	80	80	90	28	3
		28	18			2			
DEMAND	22	28	18	15	18	2	24	144	
Difference	50	2	40	40	30	20	50		

Table 11: The ninth VAM Allocation

	A	B	C	D	E	F	G	CAPACITY	Diff
Y ₁	50	85	90	30	50	60	40	17	25
	22			15	18	17	24		
Y ₂	100	83	50	70	80	80	90	28	3
		28	18			2			
DEMAND	22	28	18	15	18	2	24	144	
Difference	50	2	40	40	30	20	50		

Table 12: The initial VAM solution

	A	B	C	D	E	F	G	CAPACITY	Diff
Y ₁	50	85	90	30	50	60	40		
	22			15	18	17	24		
Y ₂	100	83	50	70	80	80	90		
		28	18			2			
DEMAND									
Difference									

$$Z = 50 * 22 + 30 * 15 + 50 * 18 + 60 * 17 + 40 * 24 + 83 * 28 + 50 * 18 + 80 * 2 = 78$$

5. Application of Modified Distribution Method to Test Optimality of Initial Solution

STEP 1

The numbers of occupied cells are $m+n-1 = 2 + 7 - 1 = 8$ implies that initial solution is non-degenerate. Thus an optimal solution can be obtained. The total transportation cost associated with this solution is #7,814,000.00

Table 13: The Initial VAM Solution

TO \ FROM	A	B	C	D	E	F	G	CAPACITY	U _i
Y ₁	50	85	90	30	50	60	40		
	22			15	18	17	24		
Y ₂	100	83	50	70	80	80	90		
		28	18			2			
DEMAND									
V _i									

STEP 2

In order to compute the values of $U_i(i=1,2)$ and $V_j(j=1,2,\dots,7)$ for each occupied cell we arbitrarily assign $V_6=0$
 Given $V_6=0$, U_1 and U_2 can be computed immediately by using the relation
 $U_i + V_j = C_{ij}$ (unit transportation cost for cell ij) for occupied cells as shown below:

Let,

$$V_6 = 0$$

$$C_{16} = U_1 + V_6 \Rightarrow U_1 = 60$$

$$C_{26} = U_2 + V_6 \Rightarrow U_2 = 80$$

$$C_{11} = U_1 + V_1 \Rightarrow V_1 = 50 - 60 = -10$$

$$C_{14} = U_1 + V_4 \Rightarrow V_4 = 30 - 60 = -30$$

$$C_{15} = U_1 + V_5 \Rightarrow V_5 = 50 - 60 = -10$$

$$C_{16} = U_1 + V_6 \Rightarrow V_6 = 60 - 60 = 0$$

$$C_{17} = U_1 + V_7 \Rightarrow V_7 = 40 - 60 = -20$$

$$C_{22} = U_2 + V_2 \Rightarrow V_2 = 83 - 80 = 3$$

$$C_{23} = U_2 + V_3 \Rightarrow V_3 = 50 - 80 = -30$$

$$C_{26} = U_2 + V_6 \Rightarrow V_6 = 80 - 80 = 0$$

STEP 3

The opportunity cost for each of the unoccupied cells is determined by using the relation
 $D_{ij} = C_{ij} - (U_i + V_j)$ and is shown below:

$$D_{12} = C_{12} - (U_1 + V_2) = 85 - (60 + 3) = 22$$

$$D_{13} = C_{13} - (U_1 + V_3) = 90 - (60 - 30) = 60$$

$$D_{21} = C_{21} - (U_2 + V_1) = 100 - (80 - 10) = 30$$

$$D_{24} = C_{24} - (U_2 + V_4) = 70 - (80 - 30) = 20$$

$$D_{25} = C_{25} - (U_2 + V_5) = 80 - (80 - 10) = 10$$

$$D_{27} = C_{27} - (U_2 + V_7) = 90 - (80 - 20) = 30$$

According to the optimality criterion for cost minimizing transportation problem, the current solution is optimal, since the opportunity cost for unoccupied cells are all positive.

6. Result and Discussion

The above transportation problem was solved using VAM and optimality test was carried out using MODI, the result obtained shows that the initial VAM solution is optimal.

The Matlab solution using Gaussian method shows that the minimum total transportation cost is #9,100,000.00

The values for the basic variables show the optimal amounts to ship over each route. The logistics manager should follow the following distribution list if want to optimize the distribution:

- Ship 22 trucks of product from Y_1 to A.
- Ship 15 trucks of product from Y_1 to D.
- Ship 18 trucks of product from Y_1 to E.
- Ship 17 trucks of product from Y_1 to F.
- Ship 24 trucks of product from Y_1 to G.
- Ship 28 trucks of product from Y_2 to B.
- Ship 18 trucks of product from Y_2 to C.
- Ship 2 trucks of product from Y_2 to F.

7. Conclusion

The transportation cost is an important element of the total cost structure for any business, the transportation problem was formulated and initial feasible solution was obtain using VAM, the solution obtained was tested for optimality using MODI.

The computational results provided the minimal total transportation cost and the values for the decision variables for optimality. Optimum solutions provided the valuable information such as analysis for industries to make optimal decisions through the use of this mathematical model (Transportation Model), the model can identify easily and efficiently plan out transportation problem.

6. References

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