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Growth And Approximation Properties Of Entire Functions

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1. Abstract

The classical growth have been characterized in terms of approximation errors for a function continuous on [-1,1] by A.R. Reddy [14], and a compact set K of positive capacity by T. Winiarski [27] with respect to maximum norm. For a nonconstant entire function $f(z) = \sum_{k=0}^{\infty} a_k z^k$ and $M(f,r) = max_{|z|=r}|f(z)|$,it is well known that the function $r \to \log M(f,r)$is indefinitely increasing convex function of $\log r$. To establish the growth of $f$precisely, R.P. Boas [2] has introduced the concept of order, defined by the number $\rho(0 \le \rho \le \infty)$:

$$\rho = \lim_{r\to\infty} \sup \frac{\log\log M(f,r)}{logr}.$$

The concept of type has been introduced to determine the relative growth of two functions of same nonzero finite order. An entire function of order ρ $(0 < \rho \le \infty)$, is said to be of type T $(0 \le T \le \infty)$ if

$$T = \lim\ sup_{r\to\infty} \frac{logM(f,r)}{r^{\rho}}.$$

If f is an entire function of infinite or zero order, the definition of type is not valid and the growth of such function can not be measured by the above concept . However, Juneja et al. [8] have introduced the concept of index-pair to study the growth of entire functions and established the relationship of (p,q)-growth of f in terms of the coefficients occurring in the Maclaurin series expansion.

2. Literature Survey

A.V. Batyrev [1], A.R. Reddy [14,15] and R.S. Varga [26] showed that the rate of convergence to zero of elements of the sequence $\{E_n(f)c\}$ of the best polynomial approximation of an entire transcendental function depends on the characteristics of its growth such as its order ρ *and type* T. For example, according to R.S. Varga [26], the fulfillment of the equality

$$\text{limsup}_{n\to\infty} \frac{nlogn}{-\log\{E_n(f)c\}} = \rho,$$

is the necessary and sufficient condition for $f(x) \in C([-1,1])$ to be restriction of the entire transcendental function f(z) of order ρ, *where* $\rho \in [0,\infty)$, *on* $[-1,1]$.

I.I. Ibragimov and N.I. Shikhaliev [6,7] , A. Giroux [4], S.B. Vakarchuk [18,19] and others [20-24] studied the limiting equalities of an analogous content, which connect the characteristic of growth of entire transcendental functions with sequences of their best polynomial approximations in some Banach spaces of analytic functions, in complex plane.

M.N.Sheremeta [16] introduced a more flexible generalized scale of growth of entire functions and opened new possibilities for studies of the connection between various generalized characteristics of growth of function and their best polynomial approximation in some Banach spaces of functions analytic in a simply connected domain (see. [16,17]).

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Vakarchuk and Zhir [25] studied the generalized order of functions analytic in the unit disc in terms of algebraic polynomial approximation errors in Banach spaces (B(p,q,m) space, Hardy space and Bergman space). Also, they obtained a Hadamard-type theorem that associates the generalized order of growth of an entire transcendental function f with the coefficients of its expansion in a Faber series.

Recently, Kumar [9] studied the L^p-approximation of entire functions over Jordan domains by using Faber polynomials and improved the various results of Sheremeta [16] and Ganti and Srivastava [5].

Following extensions of Bernstein's theorem [6,14,15,26] in analytic function theory, the focus was on harmonic polynomial approximates of real-valued (even) generalized biaxially symmetric potentials (GBASP) in $L^p\,(\Delta)\, for\ fixed\ p \geq 2$. Identified [10,11] were those that were restrictions of entire GBASP functions to an open disc Δ. In addition, the orders and types were computed. McCoy [13] improved above results by approximating GBASP in $L^p(D)$ on certain open sets D that are symmetric about the origin with Jordan boundary.

3. Objective of the Work

The main objective of the work is to study the growth and approximation properties of entire functions, and by using these properties we will study some results which will be applicable in physical sciences.

4. Proposed Work

Inspired by the work of Zeriahi [28], S.M. Einstein-Matthews and C.H. Lutterodt [3] studied the growth of entire functions in $C^{m+n}, m > 1$. They have extended the notions of (p,q)- order and (p,q)-type studied in [8] to transcendental entire functions, defined on a complete intersection algebraic variety in C^{m+n} of codimension m>1 and characterized the results in terms of an orthogonal basis in a Hilbert space $L^2(X,\mu)\ where\ \ \mu$ is a capacitary extremal measure. We are interested to extend above results in terms of polynomial approximation errors in $L^p\,(X,\mu), 0 < p \leq \infty$.

The results of McCoy [13] becomes of special interest when examining the relationship between axisymmetric potentials and solutions of more general linear elliptic partial differential equations by method of Ascent as found for Bernstein theorems on $L^\infty(\Delta)$ in [12]. We are also interested to study the results of McCoy in above direction.

5. Significance of the Work

The Euler- Poisson-Darboux equation, arising in gas dynamics, is viewed in terms of certain elliptic partial differential equation after a suitable transformation and have a variety of physical interpretations. Although, bi-axially symmetric potential theory is a well developed subject with many applications to the physical sciences it is, perhaps, not fully appreciated that certain biological problems suggest the use of this theory. The problem of steady-state differential flow through a cylindrical structure arises frequently. Not surprisingly, the physiological situation may provide motivation for solving problems and seeking techniquis that are different from those arising from purely mathematical or physical considerations. Also, these potentials play an important role in many aspects of mathematical physics, in particular to an understanding of compressible flow in the transonic region.

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