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A Study on Fixed Charge Transportation Problem with Enhanced Flow Due to New requirements in the market

VASANTH KUMAR BONIFACE, Associate Professor, Department of Mathematics, Sri Ramakrishna College of Arts and science, Coimbatore

Dr HANNAH REVATHY F. Dean, School of Science and Humanities, Sri Ramakrishna College of Arts and science, Coimbatore

ABSTRACT:

This paper deals with the Fixed Charge Transportation Problem with Enhanced Flow due to new requirements in the market. This new demand in the market, required the flow to be enhanced, compelling some of the factories to increase their production. An algorithm to find an optimal total cost for this related fixed charge transportation problem is given with a numerical example to illustrate the same.

(Key words: Fixed charge Transportation Problem, Enhanced flow)

I. INTRODUCTION

In general a transportation problem is concerned with distributing any commodity from any group of supply centres called 'Sources' to any group of receiving centres called 'Destinations' in such a way as to minimize the total distribution costs. A standard transportation model is a model which deals with a single commodity and also has equal supply and demand. When the total supply is not equal to the total demand, the solution is said to be unbalanced.

Sometimes when there is an extra demand in the market, the flow has to be increased by forcing the factories to work more and produce more. This type of problem is referred as a related standard transportation problem. In these types of models the factories where the production needs to be increased as well as the markets where this extra production would be consumed are also identified. In certain other situations where a fixed cost is increased for every origin, the problem can be formulated as 'A fixed charge transportation problem' which is an extension of the classical transportation problem.

Model Description:

This model is a fixed charge transportation problem with enhanced flow. Many real life problems come under this category because of the fixed costs and variable costs involved in the freight rates. Some of the fixed costs can be the cost of renting a vehicle, landing fees in an airport, set up costs for machines in manufacturing environment etc.

Minimize $\sum_{i\in I}\sum_{j\in J} c_{ij}\, x_{ij} + \sum_{i\in I} F_i$

Subject to

$\sum_{j\in J} x_{ij} = a_i$, $i \in I$

$\sum_{i\in I} x_{ij} = b_j$, $j \in J$

$x_{ij} \geq 0$

Where

$I = \{1,2, \ldots\ldots m\}$ is the set of origin

$J = \{1,2,\ldots\ldots\ldots n\}$ is the set of destinations

x_{ij}= amount transported from the ith origin to the jth destination

c_{ij} = the variable cost for transporting one unit from the ith origin to the jth destination.

F_i = the fixed cost associated with the ith origin.

a_i = amount available in the ith origin and

b_i = demand of the jth destination.

Here the total availability is less than the total demand and the total flow is $\sum_{j\in J} b_j$. In this situation because of the extra demand in the market, the total flow needs to be enhanced, compelling some of the factories to increase their production in order to be able to meet this extra demand. The total flow from the factories in the market is now increased by the amount of extra demand

Let P $= \sum b_j$ be the enhanced flow. The fixed charge transportation with enhanced flow is given by

Minimize $\sum_{i\in I} \sum_{j\in J} c_{ij}\, x_{ij} + \sum_{i\in I} F_i$

Subject to,

$\sum_{j\in J} x_{ij} \geq a_i$, $i \in I$

$\sum_{i\in I} x_{ij} = b_j$, $j \in J$

$\sum_{i\in I} \sum_{j\in J} x_{ij} = P$ Where $P = \sum_{j\in J} b_j$

For formulation of F_i ($i \in I$) it is assumed that F_i ($i \in I$) has p number of steps so that

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$F_i = \sum_{j=1}^{n} \delta_{ij} F_{ij}$,$i \in I$

Where δ_{ij} = 1 if $\sum_{i=j} x_{ij} > A_{il}$, $i \in I$, $l = 1,2,\ldots\ldots P$

= 0 otherwise

Here $0 = A_{i1} < A_{i2} < \ldots\ldots < A_{ip}$

$A_{i1}, A_{i2}, \ldots\ldots A_{ip} (i \in I)$ are constants and

F_{il} ($l = 1,2,\ldots\ldots$i..P,$i \in I$) are fixed costs.

In order to deal with the flow constraints $\sum_{i \in I} \sum_{j \in J} x_{ij}$ = P, a related fixed charge transportation problem is formulated by adding a fictitious factory with availability equal to (P-$\sum_{i \in I} a_i$)

The related fixed charge transportation problem is

Minimize $\sum_{i \in I'} \sum_{j \in J'} c'_{ij} x_{ij} + \sum_{i \in I'} F_i$

Subject to

$\sum_{i \in j'} y_{ij}$ = a_i', $i \in I' = I \cup \{m+1\}$

$\sum_{i \in I'} y_{ij}$ = b_j' , $j \in J'$ ………………(2)

$Y_{ij} \geq 0$; $i \in I'$, $j \in J'$

Where

$a_i' = a_i$,$i \in I$ and $a'_{m+1} = P - \sum_{i \in I} a'_i$

$b_j' = b_j$, $j \in J$

$c_{ij}' = c_{ij}$,$i \in I$, $j \in J$

c'_{m+1} J = min $c_{ij} \notin$

$i \in j$

ALGORITHM

Step 1: Starting with a basic feasible solution of the related fixed charge transportation problem with basic matrix B, calculate the fixed cost and denote it by F'(Current) where F'(Current) = $\sum_{i=1}^{m+1} F_i$ and find $c_{ij} - u_i' - v_j'$ for all $(i,j) \in B$ and denote it by (c_{ij}) where u_i',v_j' are the dual variables for i=1,2…….m+1, j=1,2,…….n

Step 2: Find $(A_{ij})' = (c_{ij})' \times (E_{ij})'$

Where $A_{ij}'$is the change in cost on introducing a non basic cell (i,j) with value $(E_{ij})'$ (for all $(i,j) \notin B$) into the basis and find F_{ij}' (Difference) = F_{ij}' (NB) – F' (Current)

Where F_{ij}' (NB) is the total fixed cost involed on introducing the non basic variable y_{ij} with value $(E_{ij})'$ into the current basis to form a new basis.Also calculate $\Delta_{ij}' = F_{ij}'$ (difference) + A_{ij}'

($\forall$ $(i,j) \notin B$). Enter the variable y_{ij} corresponding to min { Δ_{ij}/ $\Delta_{ij}<0$}

Repeat the process till all $\Delta_{ij}' \geq 0$

Step 3: Let C' be the optimal cost of related fixed charge transportation problem yielded by the basic feasible solution $\{y_{ij}'\}$

NUMERICAL EXAMPLE

Fixed charge transportation problem with enhanced flow is given below

$M_1 M_2 M_3$ a_i

	M_1	M_2	M_3	a_i
F_1	4	9	3	18
F_2	5	6	9	11
F_3	6	7	1	11
b_j	20	10	20	

The Fixed costs are

F_{11} =100, F_{12} =50, F_{13} =50

F_{21} =150, F_{22} =50, F_{23} =50

F_{31} =200, F_{32} =100, F_{33} =50

Where $F_i = \sum_{l=1}^{3} \delta_{il} F_{il}$,i=1,2,3

and $\delta_{i1} = 1$ if $\sum_{j=1}^{3} x_{ij} > 0$ for i=1,2,3

= 0 otherwise

$\delta_{i2} = 1$ if $\sum_{j=1}^{3} x_{ij} > 11$ for i=1,2,3

= 0 otherwise

$\delta_{i3} = 1$ if $\sum_{j=1}^{3} x_{ij} > 19$ for i=1,2,3

= 0 otherwise

$\sum a_i = 40$ and $\sum b_j = 50$

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$\sum a_i \neq \sum b_j$

A related fixed charge transportation problem is formulated by adding a fictitious factory with available equal to 10.

The related fixed charge transportation problem is

4	9	3	18
5	6	9	11
6	7	1	11
4	6	1	10
20	10	20	

The initial basic feasible solution to the related fixed charge transportation problem is

4 18	9	3	
5 1	6 10	9	

[6]	[7]	[1] 11	
[4] 1	[6]	[1] 9	

The optimal solution to the related fixed charge transportation problem is $\mathbf{u_i}$.

[4] 18	[9]	[3]	4
[5] 1	[6] 10	[9]	5
[6]	[7]	[1] 11	4
[4] 1	[6]	[1] 9	3
$\mathbf{v_j}$ 0	1	-3	

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The optimal solution to the related fixed charge transportation problem with enhanced flow

4 19	9	3	≥18
5 1	6 10	9	≥11
6	7	1 20	≥11
20	10	20	

The optimal transportation cost = $\sum_{i=1}^{3} c_{ij}\, x_{ij}$ = 4x19+5x1+6x10+1x20 = Rs.161 Calculate F' (current)

4 18	9	3
5 1	6 10	9
6	7	1 11
4	6	1

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1		9

$F_1 = \sum_{i=1}^{3} \delta_{1l} F_{1l}$

$= \delta_{11}F_{11} + \delta_{12}F_{12} + \delta_{13}F_{13}$

$\delta_{11} = 1, \delta_{12} = 1, \delta_{13} = 0$

$F_1 = 1x100 + 1x50 + 0x50$

$F_1 = 150$

$F_2 = \sum_{i=1}^{3} \delta_{2l} F_{2l}$

$= \delta_{21}F_{21} + \delta_{22}F_{22} + \delta_{23}F_{23}$

$\delta_{21} = 1, \delta_{22} = 0, \delta_{23} = 0$

$F_2 = 1x150 + 0x50 + 0x50$

$F_2 = 150$

$F_3 = \sum_{i=1}^{3} \delta_{3l} F_{3l}$

$= \delta_{31}F_{31} + \delta_{32}F_{32} + \delta_{33}F_{33}$

$\delta_{31} = 1, \delta_{32} = 0, \delta_{33} = 0$

$F_3 = 1x200 + 0x100 + 0x50$

$F_3 = 200$

Fixed cost F' (current) $= \sum_{i=1}^{3} F_i$

$= F_1 + F_2 + F_3$

$= 150 + 150 + 200$

$= 500$

Find $c_{ij} - u_i - v_j$ and denote it by $(c_{ij})'$

$(c_{ij})' = c_{ij} - u_i - v_j$

$(c_{12})' = 9 - 4 - 1 = 4$

$(c_{13})' = 3 - 4 + 3 = 2$

$(c_{23})' = 9 - 5 + 3 = 7$

$(c_{31})' = 6 - 4 - 0 = 2$

$(c_{32})' = 7 - 4 - 1 = 2$

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$(c_{42})' = 6 - 4 - 1 = 1$

Find $(A_{ij})' = (c_{ij})' \times (E_{ij})'$ where $(A_{ij})'$ is the change in cost that occurs on introducing a non-basic cell (i,j) with value $(E_{ij})'$.

$(E_{12})'=10$, $(E_{13})'=9$, $(E_{23})'=1$, $(E_{31})'=1$, $(E_{32})'=1$, $(E_{42})'=1$

$(A_{12})' = 4\text{x}10 = 40$

$(A_{13})' = 9\text{x}9 = 81$

$(A_{23})' = 7\text{x}1 = 7$

$(A_{31})' = 2\text{x}1 = 2$

$(A_{32})' = 2\text{x}1 = 2$

$(A_{42})' = 1\text{x}1 = 1$

Calculate F_{ij}' (NB). For the cell (1,2)

4 8	9 10	3
5 11	6	9
6	7	1 11
4 1	6	1 9

$F_1 = \sum_{i=1}^{3} \delta_{1l} F_{1l}$

$= \delta_{11}F_{11} + \delta_{12}F_{12} + \delta_{13}F_{13}$

$\delta_{11} = 1, \delta_{12} = 1, \delta_{13} = 0$

$F_1 = 1x100 + 1x50 + 0x50$

$F_1 = 150$

$F_2 = \sum_{l=1}^{3} \delta_{2l}F_{2l}$

$= \delta_{21}F_{21} + \delta_{22}F_{22} + \delta_{23}F_{23}$

$\delta_{21} = 1, \delta_{22} = 0, \delta_{23} = 0$

$F_2 = 1x150 + 0x50 + 0x50$

$F_2 = 150$

$F_3 = \sum_{l=1}^{3} \delta_{3l}F_{3l}$

$= \delta_{31}F_{31} + \delta_{32}F_{32} + \delta_{33}F_{33}$

$\delta_{31} = 1, \delta_{32} = 0, \delta_{33} = 0$

$F_3 = 1x200 + 0x100 + 0x50$

$F_3 = 200$

F_{ij} (NB) at (1,2) = $F_1 + F_2 + F_3$

= 150 + 150 + 200

= 500

Similarly,

Calculate for the cell (1,3)

$F_1 = 150$ $F_2 = 150$ $F_3 = 200$ F_{ij} (NB) at (1,3) = 500

Calculate for the cell (2,3)

$F_1 = 150$ $F_2 = 150$ $F_3 = 200$ F_{ij} (NB) at (2,3) = 500

Calculate for the cell (3,1)

$F_1 = 150$ $F_2 = 150$ $F_3 = 200$ F_{ij} (NB) at (3,1) = 500

Calculate for the cell (3,2)

$F_1 = 150$ $F_2 = 150$ $F_3 = 200$ F_{ij} (NB) at (3,2) = 500

Calculate for the cell (4,2)

$F_1 = 150$ $F_2 = 150$ $F_3 = 200$ F_{ij} (NB) at (4,2) = 500

Calculate Δ_{ij}'

$\Delta_{ij}' = F_{ij}'$ (Diff) + A_{ij}'

But F_{ij}' (Diff) = F_{ij}' (NB) – F' (current)

Δ_{ij}' = F_{ij}' (Diff) = F_{ij}' (NB) – F' (current) + A_{ij}'

VALUES OF A_{ij}', F_{ij}' (NB) AND Δ_{ij}'

(i,j)	**(1,2)**	**(1,3)**	**(2,3)**	**(3,1)**	**(3,2)**	**(4,2)**
A_{ij}'	40	81	**7**	2	2	1
F_{ij}' (NB)	500	500	500	500	500	500
Δ_{ij}'	40	81	**7**	2	2	1

As $\Delta_{ij} \geq 0 \ \forall$ (i,j) $\nexists$ B, the solution given in the above table is an optimal solution.

Let C' be the optimal cost of the related fixed charge transportation problem with enhanced flow yielded by the basic feasible solution {y_{ij}'}

C, = $\sum_{i=1} \sum_{j=1} c_{ij} x_{ij} + \sum_{i=1} F_i$

= 161 + 500

= 661

CONCLUSION

The optimal cost of the related fixed charge transportation problem is 661.

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AUTHORS PROFILE

Mr.Vasanth Kumar Boniface, Associate Professor, Department of Mathematics, Sri Ramakrishna College of Arts and science, Coimbatore.

vasanthkumar@srcas.ac.in

Dr Hannah Revathy F., Dean, School of Science and Humanities, Sri Ramakrishna College of Arts and science, Coimbatore.

hannah@srcas.ac.in

