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Dynamical System Induced By Tensor Sum and Tensor Product Operator in Measurable Function Spaces

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Abstract

Let X be a Hausdroff topological space and let $B(E)$ be the Banach algebra of all bounded linear operators on a Banach space E. Let V be a system of weights on X. In this paper we make a study of dynamical system induced by tensor sum and tensor product operator, weighted tensor sum and tensor product operator on weighted spaces of measurable functions like $MV_0(X)$ (or $MV_0(X,E)$) and $MV_b(X)$ (or $MV_b(X,E)$) respectively.

Keywords : system of weights, measurable function, weighted tensor sum operators, weighted tensor product operator, composition operators, dynamical systems, semi norm, operator valued mapping.2000 AMs subject classification : 47 B 37, 47 B 38, 47 B 07, 47 D 03,30 H 05.

1 INTRODUCTION

L^p spaces are some of the most important spaces studied in mathematics because of our abundant usefulness and applications that run across all the branches of mathematics. This paper is generalization of L^p spaces. Multiplication, composition operator, tensor sum operator, tensor product operator, weighted tensor sum and tensor product operators have been appearing in a natural way on different spaces of continuous functions, analytic functions and cross sections. For example [2,3,4,6,7] have shown that those operator on spaces of continuous and cross-

section. R.K. Singh and Manhas studied dynamical system induced by multiplication and composition operator on continuous and holomorphic function spaces[3,5,8,9,10]. We have organized this paper into four section. In section 1, preliminaries. In section 2, we characterized functions inducing tensor sum operator on weighted locally convex spaces of measurable functions. In section 3, we obtained dynamical system induced by tensor sum operators on weighted locally convex spaces of measurable functions. In section 4,dynamical system induced by weighted tensor sum operators on weighted locally convex spaces of measurable functions.
In section 5, we characterized functions inducing tensor product operator on weighted locally convex spaces of measurable functions. In section 6, we obtained dynamical system induced by tensor product operators on weighted locally convex spaces of measurable functions. In the last section, we wide up with ,dynamical system induced by weighted tensor product operators on weighted locally convex spaces of measurable functions.

2.Tensor sum operator and dynamical system in measurable function spaces

2.1 Preliminaries

Let X be a Hausdorff topological space and $M(X,E)$ be the space of all measurable function from X into E and $C(X,E)$ be the vector space of $M(X,E)$ consisting of the continuous function f from X into E. Let V be a set of non-negative upper- semi continuous family $\{\|.\|_{v,q}: v \in V, q \in cs(E)\}$ of semi norms defines a Hausdorff locally convex topology on each of these spaces. This topology will be dented by w_v and the vector spaces $MV_0(X,E)$ and $MV_b(X,E)$ endowed with w_v are called the weighted locally convex space of vector-valued continuous functions. It has a basis of closed absolutely convex neighborhoods of the origin of the form,

$$B_{v,q} = \{f \oplus g \in MV_b(X,E) \oplus MV_b(X,E): \|f \oplus g\|_{v,q} \leq 1\}$$

Also, $MV_0(X,E)$ is a closed subspace of $MV_b(X,E)$.

Let G be a topological group with e as identity, let X be a topological space and X be a topological space and $\pi : G \times X \to X$ be the continuous map such that

a. $\pi(e,x) = x$ for every $x \in X$.
b. $\pi(st,x) = \pi(s,\pi(t,x))$ for every $t,s \in G$, $x \in X$. Then the triple (G,X,π) is called a transformation group, X is a state space. If $G = (\mathbb{R},+)$ the corresponding transformation group is called a dynamical system. The transformation group (R,X,π) is known as continuous dynamical system.

If X is a Banach space and $\pi(t,\alpha x + \beta y) = \alpha\pi(t,x) + \beta\pi(t,y)$ for $t \in \mathbb{R}$, $\alpha,\beta \in \mathbb{C}$, $x,y \in X$ then $(\mathbb{R}^{+},X,\pi)$ is called a linear dynamical sysem on X. If V is a set of weights on X suct that given any $x \in X$, there is some $v \in V$ for which $v(x) > 0$. We write $V > 0$. A set V of weights on X is said to be directed upward provided for every pair $u_{1,}u_2 \in V$ and $\alpha > 0$ there exists $v \in V$ such that $\alpha u_i \leq v$ (point wise on X) for $i = 1,2$. By a system of weights, we mean a set V of weights on X with additionally satisfies $V > 0$. Let $cs(E)$ be the set f all continuous function from X into E. If V is a system of weights on X, then the pair (X,E) is called the weighted topological system. Associated with each weighted topological system (X,V), we have the weighted spaces of continuous E-valued functions defined as

$$MV_0(X,E) = \{f \oplus g \in M(X,E) \oplus M(X,E) : vq(f \oplus g) \text{vanishes at } \infty \text{ on X for each v} \in \text{V}, \text{q} \in \text{cs(E)}\}$$

$$MV_p(X,E) = \{f \oplus g \in M(X,E) \oplus M(X,E) : vq(f \oplus g) \text{ in } L^p \text{ for all v} \in \text{V}, \text{q} \in \text{cs(E)}\}$$

$$MV_b(X,E) = \{f \oplus g \in M(X,E) \oplus M(X,E) : vq\big((f \oplus g)(x)\big) \text{ is bounded in E for all v} \in \text{V}, \text{q} \in \text{cs(E)}\}$$

Let $v \in$ V, q ∈ cs(E) and $f \oplus g \in M(X,E) \oplus M(X,E)$. If we define $\|f \oplus g\|_{v,q} = (\int_X (v(x)q(f \oplus g)(x))^p d\mu)^{\frac{1}{p}}$ for all $x \in X$, then $\|.\|_v$ can be regared as a semi norm on either $MV_0(X,E) \oplus MV_0(X,E), MV_b(X,E) \oplus MV_b(X,E)$.

2.2.Functions inducing tensor sum operators on weighted spaces of measurable functions.

Theorem:2.2.1. Let $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ be a measurable functions. Then $(C_\varphi \oplus M_{\pi t})(f \oplus g)$ is a tensor sum operator for every $t \in \mathbb{R}$, $f \oplus g \in MV_0(X) \oplus MV_0(X)$ iff $V(C_\varphi \oplus M_{\pi t}) \leq V$.

Proof: First suppose $V(C_\varphi \oplus M_{\pi t}) \leq V$. Then For all v ∈ V, there exists u ∈ V such that $v|C_\varphi \oplus M_{\pi t}| \leq u$ (point wise on X). We show that $C_\varphi \oplus M_{\pi t}$ is a continuous linear operator on $MV_0(X) \oplus MV_0(X)$. Clearly, $C_\varphi \oplus M_{\pi t}$ is linear on $MV_0(X) \oplus MV_0(X)$. In order to prove the continuity of $C_\varphi \oplus M_{\pi t}$ on $MV_0(X) \oplus MV_0(X)$ it is enough to show that $C_\varphi \oplus M_{\pi t}$ is continuous at origin. For this suppose $f_\alpha \oplus g_\alpha$ be a net in $MV_0(X) \oplus MV_0(X)$ such that $P_v(f_\alpha \oplus g_\alpha) \to 0$, for every $v \in$ V.

Now,

$$P_v(C_\varphi f_\alpha \oplus M_{\pi t} g_\alpha) = P_v(C_\varphi f_\alpha) \oplus P_v(M_{\pi t} g_\alpha) \text{ for all } t \in \mathbb{R}.$$

$$= P_v(f_\alpha \circ \varphi(x)) \oplus P_v(\pi t(x) g_\alpha(x))$$

$$= (\int_X (v(x)q(f_\alpha(\varphi(x))))^p d\mu)^{\frac{1}{p}} \oplus (\int_X (v(x)q(e^{th(x)} g_\alpha(x))))^p d\mu)^{\frac{1}{p}}$$

$$= (\int_X (v(x)q(f_\alpha(x)))^p d\mu)^{\frac{1}{p}} \oplus (\int_X (v(x)q(e^{th(x)} g_\alpha(x))))^p d\mu)^{\frac{1}{p}}$$

$$= (\int_X (u(x)q(f_\alpha(x)))^p d\mu)^{\frac{1}{p}} \oplus (\int_X \left(u(x)q\big(g_\alpha(x)\big)\right)^p d\mu)^{\frac{1}{p}}$$

$$as\ t \to 0$$

$$= P_u(f_\alpha \oplus g_\alpha) \to 0$$

This proves the continuity of $C_\varphi \oplus M_{\pi t}$ at origin and hence $C_\varphi \oplus M_{\pi t}$ is continuous on $MV_0(X) \oplus MV_0(X)$.

Conversely, suppose $C_\varphi \oplus M_{\pi t}$ is continuous linear operator on $MV_0(X) \oplus MV_0(X)$.

We shall so that V$|C_\varphi \oplus M_{\pi t}| \leq V$. Let v ∈ V. Since $C_\varphi \oplus M_{\pi t}$ is continuous origin, there exist u ∈ V such that $C_\varphi \oplus M_{\pi t}(B_u) \subseteq B_v$. We claim that $v|C_\varphi \oplus M_{\pi t}| \leq 2u$. Take $x_0 \in X$ and set $u(x_0) = \varepsilon$. In case $\varepsilon > 0$, $N = \{x \in X: u(x) < 2\varepsilon\}$ is an pen neighborhood of x_0. Then there exists $f \oplus g \in MV_0(X) \oplus MV_0(X)$ such that $0 \leq f \oplus g \leq 1$, and $f \oplus g(X - N) = 0$.

Let $h = (2\varepsilon)^{-1}(f \oplus g)$. Then clearly $h \in B_u$. Since $(C_\varphi \oplus M_{\pi t})(B_u) \subseteq B_v$, we have $(C_\varphi \oplus M_{\pi t})h \in B_v$ and this yields that $v(x)|(C_\varphi \oplus M_{\pi t})(x)||h(x)| \leq 1$, for all $x \in X$. From this it follows that $v(x)|(C_\varphi \oplus M_{\pi t})(x)||f \oplus g(x)| \leq 2\varepsilon$, for all $x \in X$. Now suppose $u(x_0) = 0$ and that $v(x_0)|(C_\varphi \oplus M_{\pi t})(x_0)| > 0$ If we put that $\varepsilon = v(x_0)|(C_\varphi \oplus M_{\pi t})(x_0)|$ it is not greater than two and set , $N = \{x \in X: u(x) < \varepsilon\}$ then N would be an open neighborhood of x_0 and we could again find $f \oplus g \in MV_0(X) \oplus MV_0(X)$ such that $0 \leq f \oplus g \leq 1$, and $f \oplus g(x_0) = 1$ and $f \oplus g(X - N) = 0$.Now let $h = \varepsilon^{-1}(f \oplus g)$. Then clearly $h \in B_u$ and $(C_\varphi \oplus M_{\pi t})(h) \in B_v$. Hence $v(x)|(C_\varphi \oplus$ $M\pi t)(x)hx \leq 1$ for all $x \in X$. This implies that $vx(C\varphi \oplus M\pi t)(x)$

$| f \oplus g(x)| \leq \varepsilon$, for all $x \in X$. From this it follows that, $v(x_0)\left|(C_\varphi \oplus M_{\pi t})(x_0)\right| \leq \frac{v(x_0)\left|(C_\varphi \oplus M_{\pi t})(x_0)\right|}{2}$ which is impossible. This proves our claim and hence the proof is complete.

Now we shall characterize tensor sum operators on $MV_0(X,E) \oplus MV_0(X,E)$ induced by scalar-valued and vector valued functions.

Theorem:2.2.2. Let $\varphi : X \rightarrow X$ and $\pi_t : X \rightarrow \mathbb{C}$ be a measurable functions. Then $(C_\varphi \oplus M_{\pi t})(f \oplus g)$ is a tensor sum operator for every $t \in \mathbb{R}$, $f \oplus g \in MV_0(X,E) \oplus MV_0(X,E)$ iff $V(C_\varphi \oplus M_{\pi t}) \leq V$.

Proof Similar to proof of theorem 2.1.

Theorem:2.2.3. Let E be a(locally multiplicative convex) lmc algebra with unit e and let $\varphi : X \rightarrow X$ and $\pi_t : X \rightarrow \mathbb{C}$ be a bounded measurable function. Then $(C_\varphi \oplus M_{\pi t})(f \oplus g)$ is a tensor sum operator on $t \in \mathbb{R}$, $f \oplus g \in MV_0(X,E) \oplus MV_0(X,E)$ iff $V_p(C_\varphi \oplus M_{\pi t}) \leq V \; for \; all \; p \in P$.

Proof:

Suppose $V_p(C_\varphi \oplus M_{\pi t}) \leq V$, for all $\in P$. Then for all v ∈ V, there exists u ∈ V such that $v_p(C_\varphi \oplus M_{\pi t}) \leq u$ (point wise on X). We shall prove that the mapping $\varphi : X \rightarrow X$ gives rise to a linear transformation C_φ from $MV_0(X,E)$ itself defined as $C_\varphi f = \varphi f$ for every $f \in MV_0(X,E)$, where the product ids pointwise continuous linear operator on $MV_0(X,E)$, we shall establish the continuity of C_φ at the origin. For this, let $\{f_\alpha \oplus g_\alpha\}$ be a net in $MV_0(X,E) \oplus MV_0(X,E)$ such that for all $v \in V, p \in P$, that $P_{v,q}(f_\alpha \oplus g_\alpha) \rightarrow 0$.

Then

$P_{v,q}(C_\varphi\, f_\alpha \oplus M_{\pi t}\, g_\alpha) = P_{v,q}(C_\varphi\, f_\alpha) \oplus P_{v,q}(M_{\pi t}\, g_\alpha)$ for all t ∈ ℝ.

$$= P_{v,q}(\, f_\alpha \circ \varphi(x)) \oplus P_{v,q}(\pi t(x) g_\alpha(x))$$

$$= (\textstyle\int_X (v(x) q(\, f_\alpha(\varphi(x))))^p d\mu)^{\frac{1}{p}} \oplus (\int_X (v(x) q(e^{th(x)} g_\alpha(x))))^p d\mu)^{\frac{1}{p}}$$

$$= (\textstyle\int_X (u(x) q(\, f_\alpha(x)))^p d\mu)^{\frac{1}{p}} \oplus (\int_X \left(u(x) q(g_\alpha(x))\right))^p d\mu)^{\frac{1}{p}}$$

as $t \to 0$

$$= P_{u,q}(\, f_\alpha) \oplus P_{u,q}(\, g_\alpha)$$

$$= P_{u,q}(\, f_\alpha \oplus g_\alpha) \to 0$$

This proves that $(C_\varphi \oplus M_{\pi t})$ is continuous origin and hence a continuous linear operator on $MV_0(X,E) \oplus MV_0(X,E)$.

Remark:2.2.4

Note that if $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ be a bounded measurable complex valued (or vector-valued) functions on X then clearly $(C_\varphi \oplus M_{\pi t})\,(f_\alpha \oplus g_\alpha)$ is a tensor sum operator on

$MV_0(X) \oplus MV_0(X)$ ($or\ MV_0(X,E) \oplus MV_0(X,E)$ for any system of weights V.

If X is a system of weights generated by the characteristic functions of compact sets, then it turns out that every continuous map induces a tensor sum operators on $MV_0(X) \oplus MV_0(X)$ ($or\ MV_0(X,E) \oplus MV_0(X,E)$ for any system of weights V.

Theorem:2.2.5. Let X be a completely Hausdorff space and let $V = \{\lambda_{\chi k} : \lambda > 0\ and\ K \subset X, X$ is compact$\}$.

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i) Every bounded $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ on $MV_0(X) \oplus MV_0(X)$

ii) Every bounded $\varphi : X \to X$ and $\pi_t : X \to E$ be a lmc with jointly continuous operator induces a tensor sum operator on $C_\varphi \oplus M_{\pi t}$ on $MV_0(X,E) \oplus MV_0(X,E)$.

Proof:

Similar proof of theorem:2.3.

Corollary:2.2.6

Let X have the discreate topology and $V = \{\lambda_{\chi k} : \lambda \geq 0\ and\ K \subset X, X$ is a finite set$\}$. Then every function $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ induces a tensor sum operator $(C_\varphi \oplus M_{\pi t})(f \oplus g)$ on $MV_0(X) \oplus MV_0(X)$(or $MV_0(X,E) \oplus MV_0(X,E)$.

3. Dynamical system induced by tensor sum operators on weighted locally convex space of measurable functions

Theorem :3.1. Let U and V be an arbitrary system of weights on G and let $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ be a continuous function. Then $C_\varphi f \oplus M_{\pi t} g$ is a tensor sum operator for every $t \in \mathbb{R}$ and $f \oplus g \in MV_0(X) \oplus MV_0(X)$ iff $V(C_\varphi \oplus M_{\pi t}) \leq U$.

Proof: To show that $C_\varphi f \oplus M_{\pi t} g$is a tensor sum operator. It is enough to prove $C_\varphi f \oplus M_{\pi t} g$ is continuous at origin. Let $v \in V$ and B_v be a neighborhood of the origin in $\in MV_b(X) \oplus MV_b(X)$. Then by the given condition, there exists $u \in U$ such that $v|C_\varphi \oplus M_{\pi t}| \leq u$. Now we claim that $(C_\varphi \oplus M_{\pi t})(B_u) \subseteq B_v$, where B_u is neighborhood of the origin in $MU_b(X) \oplus MU_b(X)$ $(or\ MU_b(X,E) \oplus MU_b(X,E))$.

Let $f \oplus g \in B_u$. Then we have

$$\left\|C_\varphi f \oplus M_{\pi t} g\right\|_v = \left\|C_\varphi f(x)\right\| \oplus \|M_{\pi t} g(x)\|$$

$$= \left(\int_X (v(x)|f \circ \varphi(x)|)^p d\mu\right)^{\frac{1}{p}} \oplus \left(\int_X (v(x)|\pi t(x) g(x)|)^p d\mu\right)^{\frac{1}{p}}$$

$$= \left(\int_X (v(x)|f(\varphi(x))|)^p d\mu\right)^{\frac{1}{p}} \oplus \left(\int_X (v(x) e^{|t|\|h\|_\infty} |g(x)|)^p d\mu\right)^{\frac{1}{p}}$$

$$\leq \left(\int_X (u(x)|f(x)|)^p d\mu\right)^{\frac{1}{p}} \oplus \left(\int_X (u(x)|g(x)|)^p d\mu\right)^{\frac{1}{p}} \ as\ t \to 0$$

$$\leq \|f(x) \oplus g(x)\|$$

$$\leq \|(f \oplus g)(x)\|$$

This proves that $(C_\varphi f \oplus M_{\pi t} g) \in B_v$ and hence $C_\varphi \oplus M_{\pi t}$ is a tensor sum operator.

Corollary:3.2. Every bounded measurable function $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ induces a tensor sum operator $C_\varphi \oplus M_{\pi t}$ on $MV_b(X) \oplus MV_b(X)$ $(or\ MV_b(X,E) \oplus MV_b(X,E))$ for a system of weights V on X.

Proof:

$C_\varphi f \oplus M_{\pi t} g$ is bounded, there exists $m > 0$ such that $\left\|(C_\varphi f \oplus M_{\pi t} g)(x)\right\| \leq m$ for all $x \in X$. Let $v \in X$. Then we have $v(x)\left\|C_\varphi f \oplus M_{\pi t} g\right\| \leq mv(x)$ for all $x \in X$.

Hence by the above theorem $C_\varphi f \oplus M_{\pi t} g$ is a tensor sum operator on $MV_b(X) \oplus MV_b(X)$ $(or\ MV_b(X,E) \oplus MV_b(X,E))$.

Note:3.3. Let $h \in F_b(\mathbb{R})$. define $\pi_t : \mathbb{R} \to B(T)$ as $\pi_t(w) = e^{th(w)}$ for all $t, w \in \mathbb{R}$.

Theorem:3.4. Let $h \in F_b(\mathbb{R})$. For each $t \in \mathbb{R}$ and let $\nabla_h : \mathbb{R} \times MV_b(\mathbb{R},T) \oplus MV_b(\mathbb{R},T) \to M(\mathbb{R},T) \oplus M(\mathbb{R},T)$ be the function defined by $\nabla_h(t, f \oplus g) = C_{\varphi t} \oplus M_{\pi t}(f \oplus g)$ for all

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$t \in \mathbb{R}$ and $f \oplus g \in MV_b(\mathbb{R},T) \oplus MV_b(\mathbb{R},T)$. Then ∇_h is a dynamical system on $MV_b(\mathbb{R},T) \oplus MV_b(\mathbb{R},T)$.

Proof:

Since $C_{\varphi t} \oplus M_{\pi t}$ is a tensor sum operator on $MV_b(\mathbb{R},T) \oplus MV_b(\mathbb{R},T)$ for all $t \in \mathbb{R}$ and $f \oplus g \in MV_b(\mathbb{R},T) \oplus MV_b(\mathbb{R},T)$. We can conclude that $\nabla_h(t, f \oplus g) \in MV_b(\mathbb{R},T) \oplus MV_b(\mathbb{R},T)$. Whenever $t \in \mathbb{R}$ and $f \oplus g \in MV_b(\mathbb{R},T) \oplus MV_b(\mathbb{R},T)$. Thus ∇_h is a function from $\mathbb{R} \times MV_b(\mathbb{R},T) \oplus MV_b(\mathbb{R},T) \to M(\mathbb{R},T) \oplus M(\mathbb{R},T)$. It can be easily seen that $\nabla_h(0, f \oplus g) = f \oplus g$ and $\nabla_h(t+s, f \oplus g) = \nabla_h(t, \nabla_h(s, f \oplus g)$. In order to show that ∇_h is a dynamical system on $MV_b(\mathbb{R},T) \oplus MV_b(\mathbb{R},T)$. It is enough to show that separately continuous map. Let us first prove the continuity on ∇_h in the first argument. Let $\{t_n \to t\}$. Then $|t_n - t| \to 0$ as $n \to \infty$, we shall show that $\nabla_h(t_n, f \oplus g) \to \nabla_h(t, f \oplus g)$ in $MV_b(\mathbb{R},T) \oplus MV_b(\mathbb{R},T)$

.Let $v \to V$. Then

$$P_v(\nabla_h(t_n, f \oplus g) - \nabla_h(t, f \oplus g))_v$$

$$= P\left(((C_{\varphi t_n} \oplus M_{\pi t_n})(f \oplus g)) - ((C_{\varphi t} \oplus M_{\pi t})\,(f \oplus g))\right)_v$$

$$= \left(\int_X (v(w) q((C_{\varphi t_n}(\mathrm{w}) f(w) \ \oplus M_{\pi t_n}(w) g(w)))^p \, d\mu)^{\frac{1}{p}}\right) -$$

$$\left(\int_X (v(w) q((C_{\varphi t}(\mathrm{w}) f(w) \ \oplus M_{\pi t}(w) g(w)))^p \, d\mu)^{\frac{1}{p}}\right)$$

$$= \left(\int_X (v(w) q((C_{\varphi t_n}(w) f(w) - C_{\varphi t}(w) f(w))(M_{\pi t_n}(w) g(w)))^p \, d\mu)^{\frac{1}{p}}\right)$$

$$+ \left(\int_X (v(w) q(C_{\varphi t}(w) f(w))(M_{\pi t_n}(w) g(w) - M_{\pi t}(w) g(w)))^p \, d\mu)^{\frac{1}{p}}\right)$$

$$\leq \left(\int_X (v(w)q(f(\varphi t_n(w)) - f(\varphi t(w)(e^{|t_n|\|h\|_\infty} qg(w)))^p d\mu)^{\frac{1}{p}}\right)$$

$$+ \left(\int_X (v(w)q(f(\varphi t(w))[e^{|t_n|\|h\|_\infty} g(w) - e^{|t|\|h\|_\infty} g(w)])^p d\mu)^{\frac{1}{p}}\right)$$

. $$\to 0 \; as \; |t_n - t| \to 0.$$

Let $f_\alpha \oplus g_\alpha$ be a net in $MV_b(\mathbb{R},T) \oplus MV_b(\mathbb{R},T)$ such that $f_\alpha \oplus g_\alpha \to f \oplus g$ in $MV_b(\mathbb{R},T) \oplus MV_b(\mathbb{R},T)$. Then $q(f_\alpha \oplus g_\alpha - f \oplus g)_v \to 0$ for all $v \in V$. We shall show that $\nabla_h(t, f_\alpha \oplus$ $g\alpha \to \nabla ht, f \oplus g$ in $MVb\mathbb{R},T \oplus MVb\mathbb{R},T$.

$$P_v(\nabla_h(t, f_\alpha \oplus g_\alpha) - \nabla_h(t, f \oplus g))_v$$

$$= P\left(\left(C_{\varphi t} \oplus M_{\pi t}\right)(f_\alpha \oplus g_\alpha)(w)) - ((C_{\varphi t} \oplus M_{\pi t})\,(f \oplus g)(w))\right)_v$$

$$= P\left(\left(C_{\varphi t} f_\alpha \oplus M_{\pi t} g_\alpha\right) - \left(C_{\varphi t} f \oplus M_{\pi t} g\right)\right)_v$$

$$\leq \left(\int_X (v(w)q(f_\alpha(\varphi t(w)) - f(\varphi t(w)))e^{|t|\|h\|_\infty} q(g_\alpha(w)))^p d\mu)^{\frac{1}{p}}\right)$$

$$+ \left(\int_X (v(w)q(f(\varphi t(w)))[e^{|t|\|h\|_\infty} g_\alpha(w) - e^{|t|\|h\|_\infty} g_\alpha(w)])^p d\mu)^{\frac{1}{p}}\right)$$

$$\to 0 \; as \; f_\alpha \oplus g_\alpha - f \oplus g \to 0.$$

This proves the continuity of ∇_h is a (linear) dynamical system on the weighted space $MV_b(\mathbb{R},T)$ $\oplus MV_b(\mathbb{R},T)$.

4. Dynamical system and weighted tensor sum operator

Theorem:4.1. Let E be a locally convex Hausdorff space such that each convergent net in E is bounded. Let $\varphi: M(X, B(E))$ and $T \in M(X,X)$. Then $(C_\varphi \oplus M_{\pi t})(f \oplus g)$ is a weighted

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tensor sum operator on $MV_b(X,E) \oplus MV_b(X,E)$ iff for every $v \in V$ and $p \in cs(E)$, there exists $u \in V$ and $q \in cs(E)$ such that $v(x)P(\varphi x(w) \leq u(\pi t(x)q(x)$v for all $x \in X$ and $w \in E$.

Remark:4.2. Let $B(E)$ be the Banach algebra of all bunded linear operators on E. Then an operator-valued map $\pi_t : X \to B(E)$ defined by $\pi_t(x) = e^{th(x)}$ for all $t \in \mathbb{R}$ and $x \in X$, where $h \in M(X, B(E))$ and $\|h\|_\infty = sup\,\{\|h(x)\|: x \in X\}$. Also $\varphi t: X \to X$ is defined by $\varphi t(x) = t + x$ the self-map. Then the weighted tensor sum operator induced by φt and π_t on the spaces of $MV_0(X,E)$ and $MV_0(X,E)$.

Theorem:4.3. Let V be an arbitrary system of weights on X. Let $\nabla: \mathbb{R} \times MV_b(X,E) \oplus MV_b(X,E) \to M(X,E) \oplus M(X,E)$ be the function defined by $\nabla(t, f \oplus g) = (C_\varphi \oplus M_{\pi t})(f \oplus g)$ for all $t \in \mathbb{R}$ and $f \oplus g \in MV_b(X,E) \oplus MV_b(X,E)$. Then ∇ is a linear dynamical system if for every $v \in V$ and $p \in cs(E)$, there exists $u \in V$ and $q \in cs(E)$ such that $v(x)P(\varphi_w \oplus \pi_t)(x) \leq u(\pi t(x)q(x)$v for all $x \in X$ and $w \in E$.

Proof:

For every $t \in \mathbb{R}$ and $C_\varphi \oplus M_{\pi t}$ is a weighted tensor sum operator on $MV_b(X,E) \oplus MV_b(X,E)$. Thus it follows that,

$\nabla(t, f \oplus g) \in MV_b(X,E) \oplus MV_b(X,E)$ for all $t \in \mathbb{R}$ and $f \oplus g \in MV_b(X,E) \oplus MV_b(X,E)$.

Clearly, ∇ is linear and $\nabla(0, f \oplus g)(x) = (C_\varphi \oplus M_{\pi t})((f \oplus g)(x))$ for all $x \in X$.

$$= (C_\varphi f \oplus M_{\pi t} g)(x)$$

$$= f(\varphi(x)) \oplus e^{th(x)} g(x)$$

$$= (f \oplus g)(x)$$

Therefore $\nabla(0, f \oplus g)(x) = f \oplus g$.

Also $\nabla(t+s, f \oplus g) = \nabla(t, \nabla(s, f \oplus g))$.

Next, to show that ∇ is a linear dynamical system, it sufficient to show that ∇ is jointly continuous map[1]. Let $t_n \to t$ for all $t \in \mathbb{R}$. Then $t_n - t \to 0$ as $n \to \infty$. The remaining proof is similar to theorem:3.4.

5.Tensor product operator and dynamical system in measurable function spaces

Let X be a Hausdorff topological space and $M(X,E)$ be the space of all measurable function from X into E and $C(X,E)$ be the vector space of $M(X,E)$ consisting of the continuous function f from X into E. Let V be a set of non-negative upper- semi continuous family $\{\|.\|_{v,q} : v \in V, q \in cs(E)\}$ of semi norms defines a Hausdorff locally convex topology on each of these spaces. This topology will be dented by w_v and the vector spaces $MV_0(X,E)$ and $MV_b(X,E)$ endowed with w_v are called the weighted locally convex space of vector-valued continuous functions. It has a basis of closed absolutely convex neighborhoods of the origin of the form,

$$B_{v,q} = \{f \otimes g \in MV_b(X,E) \otimes MV_b(X,E) : \|f \otimes g\|_{v,q} \leq 1\}$$

Also, $MV_0(X,E)$ is a closed subspace of $MV_b(X,E)$.

If V is a set of weights on X suct that given any $x \in X$, there is some $v \in V$ for which $v(x) > 0$. We write $V > 0$. A set V of weights on X is said to be directed upward provided for every pair $u_{1,}u_2 \in V$ and $\alpha > 0$ there exists $v \in V$ such that $\alpha u_i \leq v$ (point wise on X) for $i = 1,2$. By a system of weights, we mean a set V of weights on X with additionally satisfies $V > 0$. Let $cs(E)$ be the set f all continuous function from X into E. If V is a system of weights on X, then the pair (X,E) is called the weighted topological system. Associated with each weighted topological system (X,V), we have the weighted spaces of continuous E-valued functions defined as

$$MV_0(X,E) = \{f \otimes g \in M(X,E) \otimes M(X,E): vq(f \otimes g) \text{vanishes at } \infty \text{ on X for each v} \in \text{V}, \text{q} \in \text{cs(E)}\}$$

$$MV_p(X,E) = \{f \otimes g \in M(X,E) \otimes M(X,E): vq(f \otimes g) \text{ in } L^p \text{ for all v} \in \text{V}, \text{q} \in \text{cs(E)}\}$$

$$MV_b(X,E) = \{f \otimes g \in M(X,E) \otimes M(X,E): vq\big((f \otimes g)(x)\big) \text{ is bounded in E for all v} \in \text{V}, \text{q} \in \text{cs(E)}\}$$

Let $v \in$ V, q $\in$ cs(E) and $f \otimes g \in M(X,E) \otimes M(X,E)$. If we define $\|f \otimes g\|_{v,q} = (\int_X (v(x)q(f \otimes g)(x))^p \, d\mu)^{\frac{1}{p}}$ for all $x \in X$, then $\|.\|_v$ can be regared as a semi norm on either $MV_0(X,E) \otimes MV_0(X,E), MV_b(X,E) \otimes MV_b(X,E)$.

5.1.Functions inducing tensor product operators on weighted spaces of measurable function

Theorem:5.1.1. Let $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ be a measurable functions. Then $(C_\varphi \otimes M_{\pi t})(f \otimes g)$ is a tensor product operator for every $t \in \mathbb{R}$, $f \otimes g \in MV_0(X) \otimes MV_0(X)$ iff $V(C_\varphi \otimes M_{\pi t}) \leq V$.

Proof: First suppose $V(C_\varphi \otimes M_{\pi t}) \leq V$. Then For all v $\in$ V, there exists u $\in$ V such that $v|C_\varphi \otimes M_{\pi t}| \leq u$ (point wise on X). We show that $C_\varphi \otimes M_{\pi t}$ is a continuous linear operator on $MV_0(X) \otimes MV_0(X)$. Clearly, $C_\varphi \otimes M_{\pi t}$ is linear on $MV_0(X) \otimes MV_0(X)$. In order to prove the continuity of $C_\varphi \otimes M_{\pi t}$ on $MV_0(X) \otimes MV_0(X)$ it is enough to show that $C_\varphi \otimes M_{\pi t}$ is continuous at origin. For this suppose $f_\alpha \otimes g_\alpha$ be a net in $MV_0(X) \otimes MV_0(X)$ such that $P_v(f_\alpha \otimes g_\alpha) \to 0$, for every $v \in$ V.

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Now,

$P_v(C_\varphi\, f_\alpha \otimes M_{\pi t}\, g_\alpha) \;=\; P_v(C_\varphi\, f_\alpha) \otimes P_v(M_{\pi t}\, g_\alpha)$ for all t ∈ ℝ.

$$= P_v(\, f_\alpha \circ \varphi(x)) \otimes P_v(\pi t(x) g_\alpha(x))$$

$$= (\int_X (v(x)q(\, f_\alpha(\varphi(x))))^p d\mu)^{\frac{1}{p}} \otimes (\int_X (v(x)q(e^{th(x)} g_\alpha(x))))^p d\mu)^{\frac{1}{p}}$$

$$= (\int_X (u(x)q(\, f_\alpha(x)))^p d\mu)^{\frac{1}{p}} \otimes (\int_X \left(u(x)q\big(g_\alpha(x)\big)\right)^p d\mu)^{\frac{1}{p}}$$

$$as\ t \to 0$$

$$= P_u(\, f_\alpha \otimes g_\alpha) \to 0$$

This proves the continuity of $C_\varphi \otimes M_{\pi t}$ at origin and hence $C_\varphi \otimes M_{\pi t}$ is continuous on $MV_0(X) \otimes MV_0(X)$.

Conversely, suppose $C_\varphi \otimes M_{\pi t}$ is continuous linear operator on $MV_0(X) \otimes MV_0(X)$.

We shall so that V$\left|C_\varphi \otimes M_{\pi t}\right| \leq V$. Let v ∈ V. Since $C_\varphi \otimes M_{\pi t}$ is continuous orgin, there exist u ∈ V such that $C_\varphi \otimes M_{\pi t}(B_u) \subseteq B_v$. We claim that $v\left|C_\varphi \otimes M_{\pi t}\right| \leq 2u$. Take $x_0 \in X$ and set $u(x_0) = \varepsilon$. In case $\varepsilon > 0$, $N = \{x \in X : u(x) < 2\varepsilon\}$ is an pen neighborhood of x_0. Then there exists $f \otimes g \in MV_0(X) \otimes MV_0(X)$ such that $0 \leq f \otimes g \leq 1$, and $f \otimes g(X - N) = 0$.

Let $h = (2\varepsilon)^{-1}(\, f \otimes g\,)$. Then clearly $h \in B_u$. Since $(\, C_\varphi \otimes M_{\pi t}\,)(\, B_u\,) \subseteq B_v$, we have $(\, C_\varphi \otimes M_{\pi t}\,)\, h \in B_v$ and this yields that $v\,(x)\left|(\, C_\varphi \otimes M_{\pi t}\,)(\, x\,)\right|\left|\, h(\, x\,)\right| \leq 1$, for all $x \in X$. From this it follows that $v(\, x\,)\left|(\, C_\varphi \otimes M_{\pi t}\,)(\, x\,)\right|\left|\, f \otimes g\,(\, x\,)\right| \leq 2\varepsilon$, for all $x \in X$. Now suppose $u(\, x_0\,) = 0$ and that $v(\, x_0)\left|(\, C_\varphi \otimes M_{\pi t}\,)(\, x_0\,)\right| > 0$ If we put that $\varepsilon = v(\, x_0)\left|(\, C_\varphi \otimes M_{\pi t})(\, x_0)\right|$ its not greater than two and set , $N = \{x \in X : u(x) < \varepsilon\}$ then N would be an open neighbourhood of x_0 and we could again find $f \otimes g \in MV_0(X) \otimes MV_0(X)$

such that $0 \leq f \otimes g \leq 1$, and $f \otimes g(x_0) = 1$ and $f \otimes g(X - N) = 0$.Now let $h = \varepsilon^{-1}(f \otimes g)$. Then clearly $h \in B_u$ and $(C_\varphi \otimes M_{\pi t})(h) \in B_v$. Hence $v(x)\left|(C_\varphi \otimes M_{\pi t})(x)\right|\left|h(x)\right| \leq 1$ for all $x \in X$. This implies that $v(x)\left|(C_\varphi \otimes M_{\pi t})(x)\right|$

$\left|f \otimes g(x)\right| \leq \varepsilon$, for all $x \in X$. From this it follows that, $v(x_0)\left|(C_\varphi \otimes M_{\pi t})(x_0)\right| \leq \frac{v(x_0)\left|(C_\varphi \oplus M_{\pi t})(x_0)\right|}{2}$ which is impossible. This proves our claim and hence the proof is complete.

Now we shall characterize tensor product operators on $MV_0(X,E) \otimes MV_0(X,E)$ induced by scalar-valued and vector valued functions.

Theorem:5.1.2. Let $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ be a measurable functions. Then $(C_\varphi \otimes M_{\pi t})(f \otimes g)$ is a tensor product operator for every $t \in \mathbb{R}$, $f \otimes g \in MV_0(X,E) \otimes MV_0(X,E)$ iff $V(C_\varphi \otimes M_{\pi t}) \leq V$.

Proof Similar to proof of theorem 2.1.

Theorem:5.1.3. Let E be a (locally multiplicative convex) lmc algebra with unit e and let $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ be a bounded measurable function. Then $(C_\varphi \otimes M_{\pi t})(f \otimes g)$ is a tensor product operator on $t \in \mathbb{R}$, $f \otimes g \in MV_0(X,E) \otimes MV_0(X,E)$ iff $V_p(C_\varphi \otimes M_{\pi t}) \leq V \; for \; all \; p \in P$.

Remark:5.1.4

Note that if $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ be a bounded measurable complex valued (or vector-valued) functions on X then clearly $(C_\varphi \otimes M_{\pi t})(f_\alpha \otimes g_\alpha)$ is a tensor product operator on

$MV_0(X) \otimes MV_0(X)$ ($or \; MV_0(X,E) \otimes MV_0(X,E)$ for any system of weights V.

If X is a system of weights generated by the characteristic functions of compact sets, then it turns out that every continuous map induces a tensor product operators on $MV_0(X) \otimes MV_0(X)$ ($or\ MV_0(X,E) \otimes MV_0(X,E)$ for any system of weights V.

Theorem:5.1.5. Let X be a completely Hausdorff space and let $V = \{\lambda_{\chi k} : \lambda > 0\ and\ K \subset X, X$ is compact$\}$.

i) Every bounded $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ on $MV_0(X) \otimes MV_0(X)$

ii) Every bounded $\varphi : X \to X$ and $\pi_t : X \to E$ be a lmc with jointly continuous operator induces a tensor product operator on $C_\varphi \otimes M_{\pi t}$ on $MV_0(X,E) \otimes MV_0(X,E)$.

Proof:

Similar proof of theorem:2.3.

Corollary:5.1.6

Let X have the discreate topology and $V = \{\lambda_{\chi k} : \lambda \geq 0\ and\ K \subset X, X$ is a finite set$\}$. Then every function $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ induces a tensor product operator $(C_\varphi \otimes M_{\pi t})(f \otimes g)$ on $MV_0(X) \otimes MV_0(X)$(or $MV_0(X,E) \otimes MV_0(X,E)$.

6.Dynamical system induced by tensor product operators on weighted locally convex space of measurable functions

Theorem :6.1. Let U and V be an arbitrary system of weights on G and let $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ be a continuous function. Then $C_\varphi f \otimes M_{\pi t} g$ is a tensor product operator for every $t \in \mathbb{R}$ and $f \otimes g \in MV_0(X) \otimes MV_0(X)$ iff $V(C_\varphi \otimes M_{\pi t}) \leq U$.

Proof: To show that $C_\varphi f \otimes M_{\pi t} g$ is a tensor product operator. It is enough to prove $C_\varphi f \otimes M_{\pi t} g$ is continuous at origin. Let $v \in V$ and B_v be a Neighborhood of the origin in $\in$

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$MV_b(X) \otimes MV_b(X)$. Then by the given condition, there exists $u \in U$ such that $v\left|C_\varphi \otimes M_{\pi t}\right| \leq u$. Now we claim that $(C_\varphi \otimes M_{\pi t})(B_u) \subseteq B_v$, where B_u is neighborhood of the origin in $MU_b(X) \otimes MU_b(X)\ (or\ MU_b(X,E) \otimes MU_b(X,E))$.

Let $f \otimes g \in B_u$. Then we have

$$\left\|C_\varphi f \otimes M_{\pi t}g\right\|_{\boldsymbol{v}} = \left\|C_\varphi f(x)\right\| \|M_{\pi t}g(x)\|$$

$$= \left(\int_X (v(x)|f \circ \varphi(x)|)^p d\mu\right)^{\frac{1}{p}} \otimes \left(\int_X (v(x)|\pi t(x) g(x)|)^p d\mu\right)^{\frac{1}{p}}$$

$$= \left(\int_X (v(x)|f(\varphi(x))|)^p d\mu\right)^{\frac{1}{p}} \otimes \left(\int_X (v(x)e^{|t|\|h\|_\infty}|g(x)|)^p d\mu\right)^{\frac{1}{p}}$$

$$\leq \left(\int_X (u(x)|f(x)|)^p d\mu\right)^{\frac{1}{p}} \otimes \left(\int_X (u(x)|g(x)|)^p d\mu\right)^{\frac{1}{p}}\ as\ t \to 0$$

$$\leq \|(f \otimes g)(x)\|$$

This proves that $(C_\varphi f \otimes M_{\pi t} g) \in B_v$ and hence $C_\varphi \otimes M_{\pi t}$ is a tensor product operator.

Corollary:6.2. Every bounded measurable function $\varphi : X \to X$ and $\pi_t : X \to \mathbb{C}$ induces a tensor sum operator $C_\varphi \otimes M_{\pi t}$ on $MV_b(X) \otimes MV_b(X)\ (or\ MV_b(X,E) \otimes MV_b(X,E))$ for a system of weights V on X.

Proof:

$C_\varphi f \otimes M_{\pi t} g$ is bounded, there exists $m > 0$ such that $\left\|(C_\varphi f \otimes M_{\pi t} g)(x)\right\| \leq m$ for all $x \in X$. Let $v \in X$. Then we have $v(x)\left\|C_\varphi f \otimes M_{\pi t} g\right\| \leq mv(x)$ for all $x \in X$.

Hence by the above theorem $C_\varphi f \otimes M_{\pi t} g$ is a tensor product operator on $MV_b(X) \otimes MV_b(X)\ (or\ MV_b(X,E) \otimes MV_b(X,E))$.

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Note:6.3. Let $h \in F_b(\mathbb{R})$. define $\pi_t : \mathbb{R} \to B(T)$ as $\pi_t(w) = e^{th(w)}$ for all $t, w \in \mathbb{R}$.

Theorem:6.4. Let $h \in F_b(\mathbb{R})$. For each $t \in \mathbb{R}$ and let $\nabla_h: \mathbb{R} \times MV_b(\mathbb{R}, T) \otimes MV_b(\mathbb{R}, T) \to M(\mathbb{R}, T) \otimes M(\mathbb{R}, T)$ be the function defined by $\nabla_h(t, f \otimes g) = C_{\varphi t} \otimes M_{\pi t}(f \otimes g)$ for all $t \in \mathbb{R}$ and $f \otimes g \in MV_b(\mathbb{R}, T) \otimes MV_b(\mathbb{R}, T)$. Then ∇_h is a dynamical system on $MV_b(\mathbb{R}, T) \otimes MV_b(\mathbb{R}, T)$.

7. Dynamical system and weighted tensor product operator

Theorem:7.1. Let E be a locally convex Hausdorff space such that each convergent net in E is bounded. Let φ: $M(X, B(E))$ and $T \in M(X, X)$. Then $(C_\varphi \otimes M_{\pi t})(f \otimes g)$ is a weighted tensor product operator on $MV_b(X, E) \otimes MV_b(X, E)$ iff for every $v \in V$ and $p \in cs(E)$, there exists $u \in V$ and $q \in cs(E)$ such that $v(x)P(\varphi x(w) \leq u(\pi t(x)q(x)$v for all $x \in X$ and $w \in E$.

Remark:7.2. Let $B(E)$ be the Banach algebra of all bunded linear operators on E. Then an operator-valued map $\pi_t : X \to B(E)$ defined by $\pi_t(x) = e^{th(x)}$ for all $t \in \mathbb{R}$ and $x \in X$, where $h \in M(X, B(E))$ and $\|h\|_\infty = sup\,\{\|h(x)\|: x \in X\}$. Also $\varphi t: X \to X$ is defined by $\varphi t(x) = t + x$ the self-map. Then the weighted tensor product operator induced by φt and π_t on the spaces of $MV_0(X, E)$ and $MV_0(X, E)$.

Theorem:7.3. Let V be an arbitrary system of weights on X. Let $\nabla: \mathbb{R} \times MV_b(X, E) \otimes MV_b(X, E) \to M(X, E) \otimes M(X, E)$ be the function defined by $\nabla(t, f \otimes g) = (C_\varphi \otimes M_{\pi t})(f \otimes g)$ for all $t \in \mathbb{R}$ and $f \otimes g \in MV_b(X, E) \otimes MV_b(X, E)$. Then ∇ is a linear dynamical system if for every $v \in V$ and $p \in cs(E)$, there exists $u \in V$ and $q \in cs(E)$ such that $v(x)P(\varphi_w \otimes \pi_t)(x) \leq u(\pi t(x)q(x)$v for all $x \in X$ and $w \in E$.

Proof:

For every $t \in \mathbb{R}$ and $C_\varphi \otimes M_{\pi t}$ is a weighted tensor product operator on $MV_b(X,E) \otimes MV_b(X,E)$. Thus it follows that,

$\nabla(t, f \otimes g) \in MV_b(X,E) \otimes MV_b(X,E)$ for all $t \in \mathbb{R}$ and $f \otimes g \in MV_b(X,E) \otimes MV_b(X,E)$.

Clearly, ∇ is linear and $\nabla(0, f \otimes g)(x) = (C_\varphi \otimes M_{\pi t})((f \otimes g)(x))$ for all $x \in X$.

$$= (C_\varphi f \otimes M_{\pi t} g)(x)$$

$$= f(\varphi(x)) \otimes e^{th(x)} g(x)$$

$$= (f \otimes g)(x)$$

Therefore $\nabla(0, f \otimes g)(x) = f \otimes g$.

Also $\nabla(t+s, f \otimes g) = \nabla(t, \nabla(s, f \otimes g))$.

Next, to show that ∇ is a linear dynamical system, it sufficient to show that ∇ is jointly continuous map[1]. Let $t_n \to t$ for all $t \in \mathbb{R}$. Then $t_n - t \to 0$ as $n \to \infty$. The remaining proof is similar to theorem:3.4.

REFERENCES

[1] P. Chernoff and Marsden, on continuity and smoothness of group actions, Bull. Amer. Math.Soc., 76(1970), 1044-1049.
[2] J. S. Manhas, Weighted composition operators on weighted locally convex spaces of analytic functions, Southeast Asian Ball. Math. 29(2005), no.1, 127-140.
[3] J.H. Shpiro, composition operator and classical function theory, Universitext: Tracts in Mathematics, Springer, New York, Ny, USA, 1993 hpiro, composition operator and classical
[4] R. K. Singh and J. S. Manhas, composition operator on function spaces, North Holland Mathematics Studies, 1993.
[5] R. K. Singh, and J. S. Manhas, Multiplication operators and dynamical systems. Journal of Austral. Math. Soc., (Series A), 53, 92-102, 1992.

[6] R. K. Singh, and J. S. Manhas, Composition operators on weighted spaces of vector-valued continuous functions, Acta. Sci. Math, 1988.

[7] R. K. Singh, and R. David Chandra Kumar, Weighted composition operators on functional spaces. Bull. Aust. Math. Soc., vol 31, 117-126, 1985.

[8] R. K. Singh, and J. S. Manhas, Multiplication operators on weighted spaces of vector-valued continuous functions. J. Aust. Math. Soc., (Series A) vol 50, 98-107, 1991.

[9] R. K. Singh, and J. S. Manhas, weighted composition operators and weighted space of cross-section. Acta scientiarum mathematicarum, Vol 59, no-3-4, 1994.

[10] R. K. Singh, and J. S. Manhas, Composition operators and Multiplication operators on weighted spaces of analytic functions, research article, international journal of mathematics and mathematical sciences, vol 2007.

[11] Senthil kumar and P.Chandra Kala, Operator and Dynamical system in measurable function spaces, Global journal of mathematical sciences: theory of practical, vol 4, no. 2(2012), pp 107-104.

[12] Senthil kumar and P.Chandra Kala, Tensor Sum and Dynamical systems, Acta Mathematica Scientia.

[13] Senthil kumar, P.Chandra Kala and T.Prasad, Tensor producd operators induce Dynamical systems on weighted locally convex space, Jordan Journal of Mathematics and statistics(JJMs) 6(3), 2013, pp.169-181.