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Analysis Of A Network Queue Model Comprised Of Parallel Channels Centrally Linked With A Common Server

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ABSTRACT: The present paper considers a network queue model in which a common channel is centrally linked in series with each of two channels such that one consists of two parallel sub-channels and other consists of three parallel sub channels. The steady state analysis of queue model has been investigated under the poison- stream. The generating function technique and laws of calculus has been used to find various queue characteristics like mean queue size, variance of queue length etc.

KEY- WORDS: Steady state behaviour, Poison stream, Generating function technique, Average-waiting time, Mean queue length, Parallel channels.

INTRODUCTION: Queuing theory is the mathematical study of waiting lines or queues. In queuing theory the term customers is used, whether referring to people or things, in correlating such variables as how customers arrive, how service meets their requirements, average service time and idle time etc. Whenever customers arrive at a service facility, some of them have to wait before they receive the desired service. It means that the customer has to wait for his/her turn, may be in a line. Customers arrive at a service facility with several queues, each with one server Queuing theory is mainly seen as a branch of applied probability theory. Its applications are in different fields, e.g. communication networks, computer systems etc. For this area there exists a huge body of publications.

Queuing theory has its beginning in the research work of a Danish engineer Erlang, A. K., when Erlang applied this theory extensively to study the behavior of telephone networks.

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Jackson [1954]studied the behaviour of a queuing system containing phase type service. O'Brien[1954] studied the transient solution of a queue model comprising of two queues in series in which the service parameter depends upon their queue lengths. Stephan[1958] discussed two queues under pre-emptive priority with poisson arrivals and service rate. Maggu[1970] introduced the concept of bitendom in theory of queues which corresponds to a practical situation arises in production concern. Later on this idea was developed by various authors with different modifications in assumptions. Matoori Towfigh and Singh T.P[1989] study a network queue model consisting two bi serial channels linked with a common server. Singh T.P. [1994] studied the Transient analysis of feedback queue model under service parameter constraint.

Recently,Singh T.P and Kumar Vinod et.al [2005]studied the transient behaviour of a queuing network with parallel bi serial queues. Further Singh T.P et al [2006] studied steady state behaviour of a queue model comprised of two subsystems with biserial channels linked with a common channel. Later on Gupta Deepak, Singh T.P et al [2007] studied a network queue model comprised of biserial channel linked with a common server. Singh T.P, Kusum, Deepak Gupta[2010] introduced the Feed back Queue Model Assumed Service Rate Proportional to Queue Number. Gupta Deepak, Naveen Gulati [2011]studied the steady state behaviour of a network queue model with biserial and parallel channels linked wiyh a common server. Meena Gupta, Deepak Gupta [2012] studied the concept of steady state solutions of multiple parallel channels in series and non-serial multiple parallel channels both in balking and reneging.

PRACTICAL SITUATION: Many practical situation of the model arise in industries, banking systems, railway, supermarkets, hospitals etc. For example: In a hospital, suppose that there are three sections. First is slip cutting counter and also there are two sub-sections in this section. One is for males and is second for females. Second section is the waiting section. This section is common for both males and females. In this section the patient wait for their turn. Third section is of doctors cabins. There are three sub-sections in this section. One is of dentist, second is of skin specialists and third is of eye specialists. Now the patients who want to consult dentist will go to the dentist and leaves the system. Similarly, the skin patients will go to the skin specialists and leaves the system and the eye patients will go to the eyes specialists and leaves the system.

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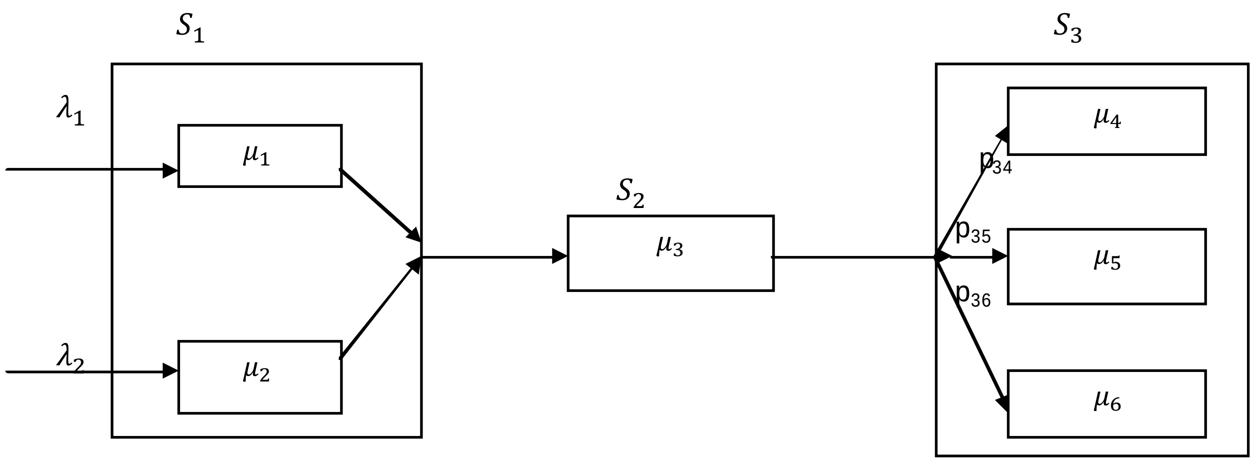
THE PROBLEM: The entire queue model is comprised of three service channels S_1,$S_2$and S_3. Where subsystem S_1 consists of two parallel service channels s_{11} and s_{12} and S_3 contains three parallel channels s_{31}, s_{32} and s_{33}.$S_1$and S_3 are linked with a common server S_2 in between both. The service time at S_{ij} are distributed exponentially. For convenience, we assume the service rate μ_1,μ_2, μ_3,μ_4, μ_5,μ_6.for μ_{ij} at S_{ij} respectively. The customers initially join S_{ij} under poisson assumptions and the mean rate λ_1,λ_2 respectively. Queues are formed in the front of the service channels $S_{11}$and S_{12} if they are busy. Customers coming at the rate λ_1 at S_{11} will go to the network of servers $S_{11} \rightarrow S_2$. Further customers coming at the rate λ_2 at S_{12} will go to the network of servers $S_{12} \rightarrow S_2$. After that the customer will go from the server S_2 either to the sub-server S_{31} with the probability p_{34} or to the sub-server S_{32} with the probability p_{35} or to the sub-server S_{33} with the probability p_{36} such that p_{34}+p_{35}+p_{36} = 1.

MATHEMATICAL ANALYSIS: Define $P_{n_1,n_2,n_3,n_4,n_5,n_6}$ the probability ,there are n_1 units wating in the queue Q_1 in front of S_{11} , n_2 units waiting in the queue Q_2 in front of S_{12} , n_3 units waiting in the queue Q_3 in front of S_2,and n_4 units wating in the queue Q_4 in front of S_{31} , n_5 units waiting in the queue Q_5 in front of S_{32}, n_6 units waiting in the queue Q_6 in front of S_{33}. In each case the waiting includes a unit in service ,if any (n_1,n_2,n_3, n_4,n_5,n_6 > 0)



Differential difference equation in transient form:

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$P_{n_1,n_2,n_3,n_4,n_5,n_6}$(t)=−$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6)$
$P_{n_1,n_2,n_3,n_4,n_5,n_6}$(t)+ λ_1 $P_{n_1-1,n_2,n_3,n_4,n_5n_6}$(t)+ $\lambda_2 P_{n_1,n_2-1\,n_3n_4,n_5,n_6}$(t)+$\mu_1(n_1$+1)
$P_{n_1+1,n_2,n_3-1,n_4n_5n_6}$(t)+ $\mu_2(n_2$+1)$P_{n_1,n_2+1n_3-1,n_4n_5,n_6}$(t)
+ $\mu_3(n_3$+1)$P_{n_1,n_2n_3+1,n_4-1n_5,n_6}(t)p_{34}$+$\mu_3(n_3$+1)$P_{n_1,n_2n_3+1,n_4n_5-1,n_6}(t)p_{35}$+$\mu_3(n_3$+1)
$P_{n_1,n_2n_3+1,n_4n_5n_6-1}(t)p_{36}$+ $\mu_4(n_4$+1)$P_{n_1,n_2n_3n_4+1n_5,n_6}$(t)+ $\mu_5(n_5$+1)$P_{n_1,n_2n_3n_4n_5+1,n_6}$(t)+ $\mu_6(n_6$+ 1)$P_{n_1,n_2n_3n_4n_5n_6+1}$(t)

Equation in steady state form:

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6)$
$P_{n_1,n_2,n_3,n_4,n_5,n_6} = \lambda_1 P_{n_1-1,n_2,n_3,n_4,n_5,n_6} + \lambda_2 P_{n_1,n_2-1,n_3,n_4,n_5,n_6} + \mu_1 P_{n_1+1,n_2,n_3-1,n_4,n_5,n_6} +$
$\mu_2 P_{n_1,n_2+1,n_3-1,n_4,n_5,n_6} + \mu_3 P_{n_1,n_2n_3+1,n_4-1,n_5,n_6} p_{34} + \mu_3 P_{n_1,n_2,n_3+1,n_4,n_5-1,n_6} p_{35} +$
$\mu_3 P_{n_1,n_2,n_3+1,n_4,n_5,n_6-1} p_{36} + \mu_4 P_{n_1,n_2,n_3,n_4+1,n_5,n_6} + \mu_5 P_{n_1,n_2,n_3,n_4,n_5+1,n_6} + \mu_6 P_{n_1,n_2,n_3,n_4,n_5,n_6+1}$
$(n_1, n_2, n_3, n_4, n_5, n_6 > 0)$ (1)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6)$ $P_{0,n_2,n_3,n_4,n_5,n_6}$
$= \lambda_2 P_{0,n_2-1,n_3,n_4,n_5,n_6} + \mu_1 P_{1,n_2,n_3-1,n_4,n_5,n_6} + \mu_2 P_{0,n_2+1,n_3-1,n_4,n_5,n_6} + \mu_3 P_{0,n_2n_3+1,n_4-1,n_5,n_6} p_{34} +$
$\mu_3 P_{0,n_2,n_3+1,n_4,n_5-1,n_6} p_{35} + \mu_3 P_{0,n_2,n_3+1,n_4,n_5,n_6-1} p_{36} + \mu_4 P_{0,n_2,n_3,n_4+1,n_5,n_6} + \mu_5 P_{0,n_2,n_3,n_4,n_5+1,n_6}$
$+ \mu_6 P_{0,n_2,n_3,n_4,n_5,n_6+1}$ $(n_1 = 0, n_2, n_3, n_4, n_5, n_6 > 0)$ (2)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_3 + \mu_4 + \mu_5 + \mu_6)$ $P_{n_1,0,n_3,n_4,n_5,n_6}$
$= \lambda_1 P_{n_1-1,0,n_3,n_4,n_5,n_6} + \mu_1 P_{n_1+1,0,n_3-1,n_4,n_5,n_6} + \mu_2 P_{n_1,1,n_3-1,n_4,n_5,n_6} + \mu_3 P_{n_1,0,n_3+1,n_4-1,n_5,n_6} p_{34} +$
$\mu_3 P_{n_1,0,n_3+1,n_4,n_5-1,n_6} p_{35} + \mu_3 P_{n_1,0,n_3+1,n_4,n_5,n_6-1} p_{36} + \mu_4 P_{n_1 0,n_3,n_4+1,n_5,n_6} + \mu_5 P_{n_1,0,n_3,n_4,n_5+1,n_6} +$
$\mu_6 P_{n_1,0,n_3,n_4,n_5,n_6+1}$ $(n_2 = 0, n_1, n_3, n_4, n_5, n_6 > 0)$ (3)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_4 + \mu_5 + \mu_6)$
$P_{n_1,n_2 0,n_4,n_5,n_6} = \lambda_1 P_{n_1-1,n_2,0,n_4,n_5,n_6} + \lambda_2 P_{n_1,n_2-1,0,n_4,n_5,n_6} + \mu_3 P_{n_1,n_2,1,n_4-1,n_5,n_6} p_{34} +$
$\mu_3 P_{n_1,n_2,1,n_4,n_5-1,n_6} p_{35} + \mu_3 P_{n_1,n_2,1,n_4,n_5,n_6-1} p_{36} + \mu_4 P_{n_1,n_2,0,n_4+1,n_5,n_6} + \mu_5 P_{n_1,n_2,0,n_4,n_5+1,n_6} +$
$\mu_6 P_{n_1,n_2,0,n_4,n_5,n_6+1}$ (4)

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$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_5 + \mu_6)$
$P_{n_1,n_2,n_3,0,n_5,n_6} = \lambda_1 P_{n_1-1,n_2,n_3,0,n_5,n_6} + \lambda_2 P_{n_1,n_2-1 n_3,0,n_5,n_6} + \mu_1 P_{n_1+1,n_2,n_3-1,0,n_5,n_6} +$
$\mu_2 P_{n_1,n_2+1,n_3-1,0,n_5,n_6} + \mu_3 P_{n_1,n_2,n_3+1,0,n_5-1,n_6} p_{35} + \mu_3 P_{n_1,n_2,n_3+1,0,n_5,n_6-1} p_{36} + \mu_4 P_{n_1,n_2,n_3,1,n_5,n_6}$
$+ \mu_5 P_{n_1,n_2,n_3,0,n_5+1,n_6} + \mu_6 P_{n_1,n_2,n_3,0,n_5,n_6+1}$
(5)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_6)$
$P_{n_1,n_2,n_3,n_4,0,n_6} = \lambda_1 P_{n_1-1,n_2,n_3,n_4,0,n_6} + \lambda_2 P_{n_1,n_2-1,n_3,n_4,0,n_6} + \mu_1 P_{n_1+1,n_2,n_3-1,n_4,0,n_6} +$
$\mu_2 P_{n_1,n_2+1,n_3-1,n_4,0,n_6} + \mu_3 P_{n_1,n_2 n_3+1,n_4-1,0,n_6} p_{34} + \mu_3 P_{n_1,n_2,n_3+1,n_4,0,n_6-1} p_{36} +$
$\mu_4 P_{n_1,n_2,n_3,n_4+1,0,n_6} + \mu_5 P_{n_1,n_2,n_3,n_4,1,n_6} + \mu_6 P_{n_1,n_2,n_3,n_4,0,n_6+1}$ (6)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5)$
$P_{n_1,n_2,n_3,n_4,n_5,0} = \lambda_1 P_{n_1-1,n_2,n_3,n_4,n_5,0} + \lambda_2 P_{n_1,n_2-1,n_3,n_4,n_5,0} + \mu_1 P_{n_1+1,n_2,n_3-1,n_4,n_5,0} +$
$\mu_2 P_{n_1,n_2+1,n_3-1,n_4,n_5,0} + \mu_3 P_{n_1,n_2 n_3+1,n_4-1,n_5,0}\, p_{34} + \mu_3 P_{n_1,n_2,n_3+1,n_4,n_5-1,0}\, p_{35} +$
$\mu_4 P_{n_1,n_2,n_3,n_4+1,n_5,0} + \mu_5 P_{n_1,n_2,n_3,n_4,n_5+1,0} + \mu_6 P_{n_1,n_2,n_3,n_4,n_5,1}$ (7)

$(\lambda_1 + \lambda_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6)$ $P_{0,0,n_3,n_4,n_5,n_6}$
$= \mu_1 P_{1,0,n_3-1,n_4,n_5 n_6} + \mu_2 P_{0,1,n_3-1,n_4,n_5,n_6} + \mu_3 P_{0,0,n_3+1,n_4-1,n_5,n_6} p_{34} + \mu_3 P_{0,0,n_3+1,n_4,n_5-1,n_6} p_{35} +$
$\mu_3 P_{0,0,n_3+1,n_4,n_5,n_6-1} p_{36} + \mu_4 P_{0,0,n_3,n_4+1,n_5,n_6} + \mu_5 P_{0,0,n_3,n_4,n_5+1,n_6} + \mu_6 P_{0,0,n_3,n_4,n_5,n_6+1}$
$(n_1, n_2 = 0, n_3, n_4, n_5, n_6 > 0)$ (8)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_4 + \mu_5 + \mu_6)$ $P_{0,n_2,0,n_4,n_5,n_6}$
$= \lambda_2 P_{0,n_2-1,0,n_4,n_5,n_6} + \mu_3 P_{0,n_2,1,n_4-1,n_5,n_6} p_{34} + \mu_3 P_{0,n_2,1,n_4,n_5-1,n_6} p_{35} + \mu_3 P_{0,n_2,1,n_4,n_5,n_6-1} p_{36} +$
$\mu_4 P_{0,n_2,0,n_4+1 n_5,n_6} + \mu_5 P_{0,n_2,0,n_4,n_5+1,n_6} + \mu_6 P_{0,n_2,0,n_4,n_5,n_6+1}$ $(n_1, n_3 =$
$0, n_2, n_4, n_5, n_6 > 0)$ (9)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_3 + \mu_5 + \mu_6)$ $P_{0,n_2,n_3 0,n_5,n_6}$
$= \lambda_2 P_{0,n_2-1,n_3 0,n_5,n_6} + \mu_1 P_{1,n_2,n_3-1,0,n_5,n_6} + \mu_2 P_{0,n_2+1,n_3-1,0,n_5,n_6} + \mu_3 P_{0,n_2,n_3+1,0,n_5-1,n_6} p_{35} +$
$\mu_3 P_{0,n_2,n_3+1,0,n_5,n_6-1} p_{36} + \mu_4 P_{0,n_2,n_3,1,n_5,n_6} + \mu_5 P_{0,n_2,n_3,0,n_5+1,n_6} + \mu_6 P_{0,n_2,n_3,0,n_5,n_6+1}$
$(n_1, n_4 = 0, n_2, n_3,, n_5, n_6 > 0)$ (10)

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$(\lambda_1 + \lambda_2 + \mu_2 + \mu_3 + \mu_4 + \mu_6)$ $P_{0,n_2,n_3,n_4 0,n_6}$
$= \lambda_2 P_{0,n_2-1,n_3,n_4,0,n_6} + \mu_1 P_{1,n_2,n_3-1,n_4,0,n_6} + \mu_2 P_{0,n_2+1,n_3-1,n_4,0,n_6} + \mu_3 P_{0,n_2 n_3+1,n_4-1,0,n_6} p_{34} ++$
$\mu_3 P_{0,n_2,n_3+1,n_4,0,n_6-1} p_{36} + \mu_4 P_{0,n_2,n_3,n_4+1,0,n_6} + \mu_5 P_{0,n_2,n_3,n_4,1,n_6} + \mu_6 P_{0,n_2,n_3,n_4,0,n_6+1}$
$(n_1, n_5 = 0, n_2, n_3, n_4, n_6 > 0)$ (11)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_3 + \mu_4 + \mu_5)$ $P_{0,n_2,n_3,n_4,n_5,0}$
$= \lambda_2 P_{0,n_2-1,n_3,n_4,n_5,0} + \mu_1 P_{1,n_2,n_3-1,n_4,n_5,0} + \mu_2 P_{0,n_2+1,n_3-1,n_4,n_5,0} + \mu_3 P_{0,n_2 n_3+1,n_4-1,n_5,0} p_{34} +$
$\mu_3 P_{0,n_2,n_3+1,n_4,n_5-1,0} p_{35} + \mu_4 P_{0,n_2,n_3,n_4+1,n_5,0} + \mu_5 P_{0,n_2,n_3,n_4,n_5+1,0} + \mu_6 P_{0,n_2,n_3,n_4,n_5,0}$
$(n_1, n_6 = 0, n_2, n_3, n_4, n_5 > 0)$ (12)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_4 + \mu_5 + \mu_6)$ $P_{n_1,0,0,n_4,n_5,n_6}$
$= \lambda_1 P_{n_1-1,0,0,n_4,n_5 n_6} + \mu_3 P_{n_1,0,1,n_4-1 n_5,n_6} p_{34} + \mu_3 P_{n_1,0,1,n_4,n_5-1,n_6} p_{35} + \mu_3 P_{n_1,0,1,n_4 n_5,n_6-1} p_{36} +$
$\mu_4 P_{n_1 0,0,n_4+1,n_5,n_6} + \mu_5 P_{n_1,0,0,n_4,n_5+1,n_6} + \mu_6 P_{n_1,0,0,n_4,n_5,n_6+1}$
$(n_2, n_3 = 0, n_1, n_4, n_5, n_6 > 0)$ (13)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_3 + \mu_5 + \mu_6)$ $P_{n_1,0,n_3 0,n_5,n_6}$
$= \lambda_1 P_{n_1-1,0,n_3,0 n_5,n_6} + \mu_1 P_{n_1+1,0,n_3-1,0,n_5,n_6} + \mu_2 P_{n_1,1,n_3-1,0,n_5,n_6} + \mu_3 P_{n_1,0,n_3+1,0,n_5-1,n_6} p_{35} +$
$\mu_3 P_{n_1,0,n_3+1,0,n_5,n_6-1} p_{36} + \mu_4 P_{n_1 0,n_3,1,n_5,n_6} + \mu_5 P_{n_1,0,n_3,0 n_5+1,n_6} + \mu_6 P_{n_1,0,n_3,0,n_5,n_6+1}$
$(n_2, n_4 = 0, n_1, n_3, n_5, n_6 > 0)$ (14)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_3 + \mu_4 + \mu_6)$ $P_{n_1,0,n_3,n_4,0,n_6}$
$= \lambda_1 P_{n_1-1,0,n_3,n_4,0,n_6} + \mu_1 P_{n_1+1,0,n_3-1,n_4,0,n_6} + \mu_2 P_{n_1,1,n_3-1,n_4,0,n_6} + \mu_3 P_{n_1,0,n_3+1,n_4-1,0,n_6} p_{34} +$
$\mu_3 P_{n_1,0,n_3+1,n_4,0,n_6-1} p_{36} + \mu_4 P_{n_1 0,n_3,n_4+1,0,n_6} + \mu_5 P_{n_1,0,n_3,n_4,1,n_6} + \mu_6 P_{n_1,0,n_3,n_4,0,n_6+1}$
$(n_2, n_5 = 0, n_1, n_3, n_4, n_6 > 0)$ (15)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_3 + \mu_4 + \mu_5)$ $P_{n_1,0,n_3,n_4,n_5,0}$
$= \lambda_1 P_{n_1-1,0,n_3,n_4,n_5,0} + \mu_1 P_{n_1+1,0,n_3-1,n_4,n_5,0} + \mu_2 P_{n_1,1,n_3-1,n_4,n_5,0} + \mu_3 P_{n_1,0,n_3+1,n_4-1,n_5,0} p_{34} +$
$\mu_3 P_{n_1,0,n_3+1,n_4,n_5-1,0} p_{35} ++ \mu_4 P_{n_1 0,n_3,n_4+1,n_5,0} + \mu_5 P_{n_1,0,n_3,n_4,n_5+1,0} + \mu_6 P_{n_1,0,n_3,n_4,n_5,1}$
$(n_2, n_6 = 0, n_1, n_3, n_4, n_5 > 0)$ (16)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_5 + \mu_6)$
$P_{n_1,n_2 0,0,n_5,n_6} = \lambda_1 P_{n_1-1,n_2,0,0,n_5,n_6} + \lambda_2 P_{n_1,n_2-1,0,0,n_5,n_6} ++ \mu_3 P_{n_1,n_2,1,0,n_5-1,n_6} p_{35} +$

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$\mu_3 P_{n_1,n_2,1,0,n_5,n_6-1} p_{36} + \mu_4 P_{n_1,n_2,0,1,n_5,n_6} + \mu_5 P_{n_1,n_2,0,0,n_5+1,n_6} + \mu_6 P_{n_1,n_2,0,0,n_5,n_6+1}$ $(n_3, n_4 = 0, n_1, n_2, n_5, n_6 > 0)$ (17)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_4 + \mu_6)$
$P_{n_1,n_2 0,n_4 0,n_6} = \lambda_1 P_{n_1-1,n_2,0,n_4,0,n_6} + \lambda_2 P_{n_1,n_2-1,0,n_4 0,n_6} + \mu_3 P_{n_1,n_2,1,n_4-1,0,n_6} p_{34} + +$
$\mu_3 P_{n_1,n_2,1,n_4,0,n_6-1} p_{36} + \mu_4 P_{n_1,n_2,0,n_4+1,0,n_6} + \mu_5 P_{n_1,n_2,0,n_4,1,n_6} + \mu_6 P_{n_1,n_2,0,n_4,0,n_6+1}$
(18)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_4 + \mu_5)$
$P_{n_1,n_2 0,n_4,n_5,0} = \lambda_1 P_{n_1-1,n_2,0,n_4,n_5,0} + \lambda_2 P_{n_1,n_2-1,0,n_4,n_5,0} + \mu_3 P_{n_1,n_2,1,n_4-1,n_5,0}\, p_{34} +$
$\mu_3 P_{n_1,n_2,1,n_4,n_5-1,0} p_{35} + \mu_4 P_{n_1,n_2,0,n_4+1,n_5,0} + \mu_5 P_{n_1,n_2,0,n_4,n_5+1,0} + \mu_6 P_{n_1,n_2,0,n_4,n_5,1}$
(19)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_6)$
$P_{n_1,n_2,n_3,0,0,n_6} = \lambda_1 P_{n_1-1,n_2,n_3,0,0,n_6} + \lambda_2 P_{n_1,n_2-1 n_3,0,0,n_6} + \mu_1 P_{n_1+1,n_2,n_3-1,0,0,n_6} +$
$\mu_2 P_{n_1,n_2+1,n_3-1,0,0,n_6} + \mu_3 P_{n_1,n_2,n_3+1,0,0,n_6-1} p_{36} + \mu_4 P_{n_1,n_2,n_3,1,0,n_6} + \mu_5 P_{n_1,n_2,n_3,0,1,n_6} +$
$\mu_6 P_{n_1,n_2,n_3,0,0,n_6+1}$ (20)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_5)$
$P_{n_1,n_2,n_3,0,n_5,0} = \lambda_1 P_{n_1-1,n_2,n_3,0,n_5,0} + \lambda_2 P_{n_1,n_2-1 n_3,0,n_5,0} + \mu_1 P_{n_1+1,n_2,n_3-1,0,n_5,0} +$
$\mu_2 P_{n_1,n_2+1,n_3-1,0,n_5,0} + \mu_3 P_{n_1,n_2,n_3+1,0,n_5-1,0} p_{35} + + \mu_4 P_{n_1,n_2,n_3,1,n_5,0} + \mu_5 P_{n_1,n_2,n_3,0,n_5+1,0} +$
$\mu_6 P_{n_1,n_2,n_3,0,n_5,1}$ (21)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3 + \mu_4)$
$P_{n_1,n_2,n_3,n_4,0,0} = \lambda_1 P_{n_1-1,n_2,n_3,n_4,0,0} + \lambda_2 P_{n_1,n_2-1,n_3,n_4,0,0} + \mu_1 P_{n_1+1,n_2,n_3-1,n_4,0,0} +$
$\mu_2 P_{n_1,n_2+1,n_3-1,n_4,0,0} + \mu_3 P_{n_1,n_2 n_3+1,n_4-1,0,0} p_{34} + \mu_4 P_{n_1,n_2,n_3,n_4+1,0,0} + \mu_5 P_{n_1,n_2,n_3,n_4,1,0} +$
$\mu_6 P_{n_1,n_2,n_3,n_4,0,1}$ (22)

$(\lambda_1 + \lambda_2 + \mu_4 + \mu_5 + \mu_6)$ $P_{0,0,0,n_4,n_5,n_6}$
$= \mu_3 P_{0,0,1,n_4-1,n_5,n_6} p_{34} + \mu_3 P_{0,0,1,n_4,n_5-1,n_6} p_{35} + \mu_3 P_{0,0,1,n_4,n_5,n_6-1} p_{36} + \mu_4 P_{0,0,0,n_4+1,n_5,n_6} +$
$\mu_5 P_{0,0,0,n_4,n_5+1,n_6} + \mu_6 P_{0,0,0,n_4,n_5,n_6+1}$ $(n_1, n_2, n_3 = 0, n_4, n_5, n_6 > 0)$
(23)

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$(\lambda_1 + \lambda_2 + \mu_3 + \mu_5 + \mu_6) \quad P_{0,0,n_3,0,n_5,n_6}$
$= \mu_1 P_{1,0,n_3-1,0,n_5 n_6} + \mu_2 P_{0,1,n_3-1,0,n_5,n_6} + \mu_3 P_{0,0,n_3+1,0,n_5-1,n_6} p_{35} + \mu_3 P_{0,0,n_3+1,0,n_5,n_6-1} p_{36} +$
$\mu_4 P_{0,0,n_3,1,n_5,n_6} + \mu_5 P_{0,0,n_3,0,n_5+1,n_6} + \mu_6 P_{0,0,n_3 0,n_5,n_6+1} \quad (n_1, n_2, n_4 = 0, n_3, n_5, n_6 > 0)$
(24)

$(\lambda_1 + \lambda_2 + \mu_3 + \mu_4 + \mu_6) \quad P_{0,0,n_3,n_4,0,n_6}$
$= \mu_1 P_{1,0,n_3-1,n_4,0,n_6} + \mu_2 P_{0,1,n_3-1,n_4,0,n_6} + \mu_3 P_{0,0,n_3+1,n_4-1,0,n_6} p_{34} + \mu_3 P_{0,0,n_3+1,n_4,0,n_6-1} p_{36} +$
$\mu_4 P_{0,0,n_3,n_4+1,0,n_6} + \mu_5 P_{0,0,n_3,n_4,1,n_6} + \mu_6 P_{0,0,n_3,n_4,0,n_6+1} \quad (n_1, n_2, n_5 = 0, n_3, n_4, n_6 > 0)$
(25)

$(\lambda_1 + \lambda_2 + \mu_3 + \mu_4 + \mu_5) \quad P_{0,0,n_3,n_4,n_5,0}$
$= \mu_1 P_{1,0,n_3-1,n_4,n_5,0} + \mu_2 P_{0,1,n_3-1,n_4,n_5,0} + \mu_3 P_{0,0,n_3+1,n_4-1,n_5,0} p_{34} + \mu_3 P_{0,0,n_3+1,n_4,n_5-1,0} p_{35} +$
$\mu_4 P_{0,0,n_3,n_4+1,n_5,0} + \mu_5 P_{0,0,n_3,n_4,n_5+1,0} + \mu_6 P_{0,0,n_3,n_4,n_5,1} \quad (n_1, n_2, n_6 = 0, n_3, n_4, n_5 > 0)$ (26)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_5 + \mu_6) \quad P_{0,n_2,0,0,n_5,n_6}$
$= \lambda_2 P_{0,n_2-1,0,0,n_5,n_6} + \mu_3 P_{0,n_2,1,0,n_5-1,n_6} p_{35} + \mu_3 P_{0,n_2,1,0,n_5,n_6-1} p_{36} + \mu_4 P_{0,n_2,0,1,n_5,n_6} +$
$\mu_5 P_{0,n_2,0,0,n_5+1,n_6} + \mu_6 P_{0,n_2,0,0,n_5,n_6+1}$
$(n_1, n_3, n_4 = 0, n_2, n_5, n_6 > 0)$ (27)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_4 + \mu_6) \quad P_{0,n_2,0,n_4,0,n_6}$
$= \lambda_2 P_{0,n_2-1,0,n_4,0,n_6} + \mu_3 P_{0,n_2,1,n_4-1,0,n_6} p_{34} + \mu_3 P_{0,n_2,1,n_4,n_5,n_6-1} p_{36} + \mu_4 P_{0,n_2,0,n_4+1,0,n_6} +$
$\mu_5 P_{0,n_2,0,n_4,1,n_6} + \mu_6 P_{0,n_2,0,n_4,0,n_6+1} \quad (n_1, n_3, n_5 = 0, n_2, n_4, n_6 > 0)$
(28)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_4 + \mu_5) \quad P_{0,n_2,0,n_4,n_5,0}$
$= \lambda_2 P_{0,n_2-1,0,n_4,n_5,0} + \mu_3 P_{0,n_2,1,n_4-1,n_5,0} p_{34} + \mu_3 P_{0,n_2,1,n_4,n_5-1,0} p_{35} + \mu_4 P_{0,n_2,0,n_4+1 n_5,0} +$
$\mu_5 P_{0,n_2,0,n_4,n_5+1,0} + \mu_6 P_{0,n_2,0,n_4,n_5,1} \quad (n_1, n_3, n_6 = 0, n_2, n_4, n_5 > 0)$
(29)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_3 + \mu_6) \quad P_{0,n_2,n_3 0,0,n_6}$
$= \lambda_2 P_{0,n_2-1,n_3,0,0,n_6} + \mu_1 P_{1,n_2,n_3-1,0,0,n_6} + \mu_2 P_{0,n_2+1,n_3-1,0,0,n_6} + \mu_3 P_{0,n_2,n_3+1,0,0,n_6-1} p_{36} +$

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$\mu_4 P_{0,n_2,n_3,1,0,n_6} + \mu_5 P_{0,n_2,n_3,0,1,n_6} + \mu_6 P_{0,n_2,n_3,0,0,n_6+1}$ $(n_1, n_4, n_5 = 0, n_2, n_3, n_6 > 0)$

(30)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_3 + \mu_5)$ $P_{0,n_2,n_3 0,n_5,0}$

$= \lambda_2 P_{0,n_2-1,n_3 0,n_5,0} + \mu_1 P_{1,n_2,n_3-1,0,n_5,0} + \mu_2 P_{0,n_2+1,n_3-1,0,n_5,0} + \mu_3 P_{0,n_2,n_3+1,0,n_5-1,0} p_{35} +$

$\mu_4 P_{0,n_2,n_3,1,n_5,0} + \mu_5 P_{0,n_2,n_3,0,n_5+1,0} + \mu_6 P_{0,n_2,n_3,0,n_5,1}$ $(n_1, n_4, n_6 = 0, n_2, n_3, , n_5 > 0)$

(31)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_3 + \mu_4)$ $P_{0,n_2,n_3,n_4 0,0}$

$= \lambda_2 P_{0,n_2-1,n_3,n_4,0,0} + \mu_1 P_{1,n_2,n_3-1,n_4,0,0} + \mu_2 P_{0,n_2+1,n_3-1,n_4,0,0} + \mu_3 P_{0,n_2 n_3+1,n_4-1,0,0}\, p_{34} +$

$\mu_4 P_{0,n_2,n_3,n_4+1,0,0} + \mu_5 P_{0,n_2,n_3,n_4,1,0} + \mu_6 P_{0,n_2,n_3,n_4,0,1}$ $(n_1, n_5, n_6 = 0, n_2, n_3, n_4 > 0)$ (32)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_5 + \mu_6)$ $P_{n_1,0,0,0,n_5,n_6}$

$= \lambda_1 P_{n_1-1,0,0,0,n_5,n_6} + \mu_3 P_{n_1,0,1,0,n_5-1,n_6} p_{35} + \mu_3 P_{n_1,0,1,0,n_5,n_6-1} p_{36} + \mu_4 P_{n_1 0,0,1,n_5,n_6} +$

$\mu_5 P_{n_1,0,0,0,n_5+1,n_6} + \mu_6 P_{n_1,0,0,0,n_5,n_6+1}$ $(n_2, n_3, n_4 = 0, n_1, n_5, n_6 > 0)$

(33)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_4 + \mu_6)$ $P_{n_1,0,0,n_4,0,n_6}$

$= \lambda_1 P_{n_1-1,0,0,n_4,0,n_6} + \mu_3 P_{n_1,0,1,n_4-1,0,n_6} p_{34} + \mu_3 P_{n_1,0,1,n_4,0,n_6-1} p_{36} + \mu_4 P_{n_1 0,0,n_4+1\,0,n_6} +$

$\mu_5 P_{n_1,0,0,n_4,1,n_6} + \mu_6 P_{n_1,0,0,n_4,0,n_6+1}$ $(n_2, n_3, n_5 = 0, n_1, n_4, n_6 > 0)$

(34)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_4 + \mu_5)$ $P_{n_1,0,0,n_4,n_5,0}$

$= \lambda_1 P_{n_1-1,0,0,n_4,n_5,0} + \mu_3 P_{n_1,0,1,n_4-1 n_5,0} p_{34} + \mu_3 P_{n_1,0,1,n_4,n_5-1,0} p_{35} + \mu_4 P_{n_1 0,0,n_4+1,n_5,0} +$

$\mu_5 P_{n_1,0,0,n_4,n_5+1,0} + \mu_6 P_{n_1,0,0,n_4,n_5,1}$ $(n_2, n_3, n_6 = 0, n_1, n_4, n_5 > 0)$

(35)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_3 + \mu_6)$ $P_{n_1,0,n_3,0,0,n_6}$

$= \lambda_1 P_{n_1-1,0,n_3,0,0,n_6} + \mu_1 P_{n_1+1,0,n_3-1,0,0,n_6} + \mu_2 P_{n_1,1,n_3-1,0,0,n_6} + \mu_3 P_{n_1,0,n_3+1,0,0,n_6-1} p_{36} +$

$\mu_4 P_{n_1 0,n_3,1,0,n_6} + \mu_5 P_{n_1,0,n_3,0,1,n_6} + \mu_6 P_{n_1,0,n_3,0,0,n_6+1}$ $(n_2, n_4, n_5 = 0, n_1, n_3, n_6 > 0)$

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$(\lambda_1 + \lambda_2 + \mu_1 + \mu_3 + \mu_5)$ $P_{n_1,0,n_3 0,n_5,0}$
$=\lambda_1 P_{n_1-1,0,n_3,0,n_5,0}+\mu_1 P_{n_1+1,0,n_3-1,0,n_5,0}+\mu_2 P_{n_1,1,n_3-1,0,n_5,0}+\mu_3 P_{n_1,0,n_3+1,0,n_5-1,0}p_{35}+$
$\mu_4 P_{n_1 0,n_3,1,n_5,0}+\mu_5 P_{n_1,0,n_3,0,n_5+1,0}+\mu_6 P_{n_1,0,n_3,0,n_5,1}$ $(n_2, n_4, n_6 = 0, n_1, n_3, n_5 > 0)$ (37)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_3 + \mu_4)$ $P_{n_1,0,n_3,n_4,0,0}$
$=\lambda_1 P_{n_1-1,0,n_3,n_4,0,0}+\mu_1 P_{n_1+1,0,n_3-1,n_4,0,0}+\mu_2 P_{n_1,1,n_3-1,n_4,0,0}+\mu_3 P_{n_1,0,n_3+1,n_4-1,0,0}p_{34}+$
$\mu_4 P_{n_1 0,n_3,n_4+1,0,0}+\mu_5 P_{n_1,0,n_3,n_4,1,0}+\mu_6 P_{n_1,0,n_3,n_4,0,1}$ $(n_2, n_5, n_6 = 0, n_1, n_3, n_4 > 0)$
(38)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_6)$
$P_{n_1,n_2 0,0,0,n_6}=\lambda_1 P_{n_1-1,n_2,0,0,0,n_6}+\lambda_2 P_{n_1,n_2-1,0,0,0,n_6}+\mu_3 P_{n_1,n_2,1,0,0,n_6-1}p_{36}+\mu_4 P_{n_1,n_2,0,1,0,n_6}+$
$\mu_5 P_{n_1,n_2,0,0,1,n_6}+\mu_6 P_{n_1,n_2,0,0,0,n_6+1}$ $(n_3, n_4, n_5 = 0, n_1, n_2, n_6 > 0)$ (39)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_5)$
$P_{n_1,n_2 0,0,n_5,0}=\lambda_1 P_{n_1-1,n_2,0,0,n_5,0}+\lambda_2 P_{n_1,n_2-1,0,0,n_5,0}+\mu_3 P_{n_1,n_2,1,0,n_5-1,0}p_{35}+\mu_4 P_{n_1,n_2,0,1,n_5,0}+$
$\mu_5 P_{n_1,n_2,0,0,n_5+1,0}+\mu_6 P_{n_1,n_2,0,0,n_5,1}$ $(n_3, n_4, n_6 = 0, n_1, n_2, n_5 > 0)$ (40)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_4)$
$P_{n_1,n_2 0,n_4 0,0}=\lambda_1 P_{n_1-1,n_2,0,n_4,0,0}+\lambda_2 P_{n_1,n_2-1,0,n_4 0,0}+\mu_3 P_{n_1,n_2,1,n_4-1,0,0}p_{34}+\mu_4 P_{n_1,n_2,0,n_4+1,0,0}+$
$\mu_5 P_{n_1,n_2,0,n_4,1,0}+\mu_6 P_{n_1,n_2,0,n_4,0,1}$ $(n_3, n_5, n_6 = 0, n_1, n_2, n_4 > 0)$ (41)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2 + \mu_3)$
$P_{n_1,n_2,n_3,0,0,0}=\lambda_1 P_{n_1-1,n_2,n_3,0,0,0}+\lambda_2 P_{n_1,n_2-1 n_3,0,0,0}+\mu_1 P_{n_1+1,n_2,n_3-1,0,0,0}+\mu_2 P_{n_1,n_2+1,n_3-1,0,0,0}+$
$\mu_4 P_{n_1,n_2,n_3,1,0,0}+\mu_5 P_{n_1,n_2,n_3,0,1,0}+\mu_6 P_{n_1,n_2,n_3,0,0,1}$ $(n_4, n_5, n_6 = 0, n_1, n_2, n_3)$ (42)

$(\lambda_1 + \lambda_2 + \mu_5 + \mu_6)$ $P_{0,0,0,0,n_5,n_6}$
$=\mu_3 P_{0,0,1,0,n_5-1,n_6}p_{35}+\mu_3 P_{0,0,1,0,n_5,n_6-1}p_{36}+\mu_4 P_{0,0,0,1,n_5,n_6}+\mu_5 P_{0,0,0,0,n_5+1,n_6}+\mu_6 P_{0,0,00,n_5,n_6+1}$
$(n_1, n_2, n_3, n_4 = 0, n_5, n_6 > 0)$

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$(\lambda_1 + \lambda_2 + \mu_4 + \mu_6)\ P_{0,0,0,n_4,0,n_6}$
$= \mu_3 P_{0,0,1,n_4-1,0,n_6} p_{34} + \mu_3 P_{0,0,1,n_4,0,n_6-1} p_{36} + \mu_4 P_{0,0,0,n_4+1,0,n_6} + \mu_5 P_{0,0,0,n_4,1,n_6} +$
$\mu_6 P_{0,0,0,n_4,0,n_6+1}\ \ (n_1, n_2, n_3, n_5 = 0, n_4, n_6 > 0)$

(44)

$(\lambda_1 + \lambda_2 + \mu_4 + \mu_5)\ P_{0,0,0,n_4,n_5,0}$
$= \mu_3 P_{0,0,1,n_4-1,n_5,0} p_{34} + \mu_3 P_{0,0,1,n_4,n_5-1,0} p_{35} + \mu_4 P_{0,0,0,n_4+1,n_5,0} + \mu_5 P_{0,0,0,n_4,n_5+1,0} +$
$\mu_6 P_{0,0,0,n_4,n_5,1}\ \ (n_1, n_2, n_3, n_6 = 0, n_4, n_5 > 0)$

(45)

$(\lambda_1 + \lambda_2 + \mu_3 + \mu_6)\ P_{0,0,n_3,0,0,n_6}$
$= \mu_1 P_{1,0,n_3-1,0,0,n_6} + \mu_2 P_{0,1,n_3-1,0,0,n_6} + \mu_3 P_{0,0,n_3+1,0,0,n_6-1} p_{36} + \mu_4 P_{0,0,n_3,1,0,n_6} + \mu_5 P_{0,0,n_3,0,1,n_6} +$
$\mu_6 P_{0,0,n_3 0,0,n_6+1}$

$(n_1, n_2, n_4, n_5 = 0, n_3, n_6 > 0)$ (46)

$(\lambda_1 + \lambda_2 + \mu_3 + \mu_5)\ P_{0,0,n_3,0,n_5,0}$
$= \mu_1 P_{1,0,n_3-1,0,n_5,0} + \mu_2 P_{0,1,n_3-1,0,n_5,0} + \mu_3 P_{0,0,n_3+1,0,n_5-1,0} p_{35} + \mu_4 P_{0,0,n_3,1,n_5,0} + \mu_5 P_{0,0,n_3,0,n_5+1,0} +$
$\mu_6 P_{0,0,n_3 0,n_5,1}$

$(n_1, n_2, n_4, n_6 = 0, n_3, n_5 > 0)$ (47)

$(\lambda_1 + \lambda_2 + \mu_3 + \mu_4)\ P_{0,0,n_3,n_4,0,0}$
$= \mu_1 P_{1,0,n_3-1,n_4,0,0} + \mu_2 P_{0,1,n_3-1,n_4,0,0} + \mu_3 P_{0,0,n_3+1,n_4-1,0,0} p_{34} + \mu_4 P_{0,0,n_3,n_4+1,0,0} + \mu_5 P_{0,0,n_3,n_4,1,0}$
$+ \mu_6 P_{0,0,n_3,n_4,0,1}$

$(n_1, n_2, n_5, n_6 = 0, n_3, n_4 > 0)$ (48)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_6)\ P_{0,n_2,0,0,0,n_6}$
$= \lambda_2 P_{0,n_2-1,0,0,0,n_6} + \mu_3 P_{0,n_2,1,0,0,n_6-1} p_{36} + \mu_4 P_{0,n_2,0,1,0,n_6} + \mu_5 P_{0,n_2,0,0,1,n_6} + \mu_6 P_{0,n_2,0,0,0,n_6+1}$

$(n_1, n_3, n_4, n_5 = 0, n_2, n_6 > 0)$ (49)

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$(\lambda_1 + \lambda_2 + \mu_2 + \mu_5)$ $P_{0,n_2,0,0,n_5,0}$
$= \lambda_2 P_{0,n_2-1,0,0,n_5,0} + \mu_3 P_{0,n_2,1,0,n_5-1,0}\, p_{35} + \mu_4 P_{0,n_2,0,1,n_5,0} + \mu_5 P_{0,n_2,0,0,n_5+1,0} + \mu_6 P_{0,n_2,0,0,n_5,1}$

$(n_1, n_3, n_4, n_6 = 0, n_2, n_5 > 0)$ (50)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_4)$ $P_{0,n_2,0,n_4,0,0}$
$= \lambda_2 P_{0,n_2-1,0,n_4,0,0} + \mu_3 P_{0,n_2,1,n_4-1,0,0}\, p_{34} + \mu_4 P_{0,n_2,0,n_4+1,0,0} + \mu_5 P_{0,n_2,0,n_4,1,0} + \mu_6 P_{0,n_2,0,n_4,0,1}$
$(n_1, n_3, n_5, n_6 = 0, n_2, n_4 > 0)$ (51)

$(\lambda_1 + \lambda_2 + \mu_2 + \mu_3)$ $P_{0,n_2,n_3 0,0,0}$
$= \lambda_2 P_{0,n_2-1,n_3,0,0,0} + \mu_1 P_{1,n_2,n_3-1,0,0,0} + \mu_2 P_{0,n_2+1,n_3-1,0,0,0} + \mu_4 P_{0,n_2,n_3,1,0,0} + \mu_5 P_{0,n_2,n_3,0,1,0} +$
$\mu_6 P_{0,n_2,n_3,0,0,1}$

$(n_1, n_4, n_5, n_6 = 0, n_2, n_3 > 0)$ (52)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_6)$ $P_{n_1,0,0,0,0,n_6}$
$= \lambda_1 P_{n_1-1,0,0,0,0,n_6} + \mu_3 P_{n_1,0,1,0,0,n_6-1} p_{36} + \mu_4 P_{n_1 0,0,1,0,n_6} + \mu_5 P_{n_1,0,0,0,1,n_6} + \mu_6 P_{n_1,0,0,0,0,n_6+1}$
$(n_2, n_3, n_4, n_5 = 0, n_1, n_6 > 0)$ (53)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_5)$ $P_{n_1,0,0,0,n_5,0}$
$= \lambda_1 P_{n_1-1,0,0,0,n_5,0} + \mu_3 P_{n_1,0,1,0,n_5-1,0}\, p_{35} + \mu_4 P_{n_1 0,0,1,n_5,0} + \mu_5 P_{n_1,0,0,0,n_5+1,0} + \mu_6 P_{n_1,0,0,0,n_5,1}$

$(n_2, n_3, n_4, n_6 = 0, n_1, n_5 > 0)$ (54)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_4)$ $P_{n_1,0,0,n_4,0,0}$
$= \lambda_1 P_{n_1-1,0,0,n_4,0,0} + \mu_3 P_{n_1,0,1,n_4-1,0,0} p_{34} + \mu_4 P_{n_1 0,0,n_4+1 0,0} + \mu_5 P_{n_1,0,0,n_4,1,0} + \mu_6 P_{n_1,0,0,n_4,0,1}$
$(n_2, n_3, n_5, n_6 = 0, n_1, n_4 > 0)$ (55)

$(\lambda_1 + \lambda_2 + \mu_1 + \mu_3)$ $P_{n_1,0,n_3,0,0,0}$
$= \lambda_1 P_{n_1-1,0,n_3,0,0,0} + \mu_1 P_{n_1+1,0,n_3-1,0,0,0} + \mu_2 P_{n_1,1,n_3-1,0,0,0} + \mu_4 P_{n_1 0,n_3,1,0,0} + \mu_5 P_{n_1,0,n_3,0,1,0} +$
$\mu_6 P_{n_1,0,n_3,0,0,1}$

$(n_2, n_4, n_5, n_6 = 0, n_1, n_3 > 0)$ (56)

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$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2)$
$P_{n_1,n_2 0,0,0,0}= \lambda_1 P_{n_1-1,n_2,0,0,0,0}+ \lambda_2 P_{n_1,n_2-1,0,0,0,0}+ \mu_4 P_{n_1,n_2,0,1,0,0}+ \mu_5 P_{n_1,n_2,0,0,1,0}+$
$\mu_6 P_{n_1,n_2,0,0,0,1}$ $(n_3, n_4, n_5 = 0, n_1, n_2, n_6 > 0)$ (57)

$(\lambda_1 + \lambda_2 + \mu_6)\ P_{0,0,0,0,0,n_6} = \mu_3 P_{0,0,1,0,0,n_6-1} p_{36}+ \mu_4 P_{0,0,0,1,0,n_6}+ \mu_5 P_{0,0,0,0,1,n_6}+ \mu_6 P_{0,0,00,0,n_6+1}$

$(n_1, n_2, n_3, n_4, n_5 = 0, n_6 > 0)$ (58)

$(\lambda_1 + \lambda_2 + \mu_5)\ P_{0,0,0,0,n_5,0} = \mu_3 P_{0,0,1,0,n_5-1,0} p_{35}+ \mu_4 P_{0,0,0,1,n_5,0}+ \mu_5 P_{0,0,0,0,n_5+1,0}+ \mu_6 P_{0,0,00,n_5,1}$

$(n_1, n_2, n_3, n_4, n_6 = 0, n_5 > 0)$ (59)

$(\lambda_1 + \lambda_2 + \mu_4)\ P_{0,0,0,n_4,0,0} = \mu_3 P_{0,0,1,n_4-1,0,0} p_{34}+ \mu_4 P_{0,0,0,n_4+1,0,0}+ \mu_5 P_{0,0,0,n_4,1,0}+ \mu_6 P_{0,0,0,n_4,0,1}$

$(n_1, n_2, n_3, n_5, n_6 = 0, n_4 > 0)$ (60)

$(\lambda_1 + \lambda_2 + \mu_3)\ P_{0,0,n_3,0,0,0}$
$= \mu_1 P_{1,0,n_3-1,0,0,0}+ \mu_2 P_{0,1,n_3-1,0,0,0}+ \mu_4 P_{0,0,n_3,1,0,0}+ \mu_5 P_{0,0,n_3,0,1,0}+ \mu_6 P_{0,0,n_3 0,0,1}$
$(n_1, n_2, n_4, n_5, n_6 = 0, n_3 > 0)$ (61)

$(\lambda_1 + \lambda_2 + \mu_2)\ P_{0,n_2,0,0,0,0} = \lambda_2 P_{0,n_2-1,0,0,0,0}+ \mu_4 P_{0,n_2,0,1,0,0}+ \mu_5 P_{0,n_2,0,0,1,0}+ \mu_6 P_{0,n_2,0,0,0,1}$

$(n_1, n_3, n_4, n_5, n_6 = 0, n_2 > 0)$ (62)

$(\lambda_1 + \lambda_2 + \mu_1)\ P_{n_1,0,0,0,0,0} = \lambda_1 P_{n_1-1,0,0,0,0,0}+ \mu_4 P_{n_1 0,0,1,0,0}+ \mu_5 P_{n_1,0,0,0,1,0}+ \mu_6 P_{n_1,0,0,0,1}$

$(n_2, n_3, n_4, n_5, n_6 = 0, n_1 > 0)$ (63)

$(\lambda_1 + \lambda_2)\ P_{0,0,0,0,0,0,0} = \mu_4 P_{0,0,0,1,0,0}+ \mu_5 P_{0,0,0,0,1,0}+ \mu_6 P_{0,0,00,0,1}$

$(n_1, n_2, n_3, n_4, n_5, n_6 = 0)$ (64)

Let us define the generating function as

F(X,Y,Z,R,S,T) =
$\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{n_3=0}^{\infty} \sum_{n_4=0}^{\infty} \sum_{n_5=0}^{\infty} \sum_{n_6=0}^{\infty} P_{n_1,n_2,n_3,n_4,n_5,n_6} X^{n_1} Y^{n_2} Z^{n_3} R^{n_4} S^{n_5} T^{n_6}$

(65)

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Where $|X|, |Y|, |Z|, |R|, |S|, |T| = 1$

Also we define partial generating functions as

$$F_{n_2,n_3,n_4,n_5,n_6}(X) = \sum_{n_1=0}^{\infty} P_{n_1,n_2,n_3,n_4,n_5,n_6} X^{n_1} \quad (66)$$

$$F_{n_3,n_4,n_5,n_6}(X, Y) = \sum_{n_2=0}^{\infty} P_{n_2,n_3,n_4,n_5,n_6}(X) Y^{n_2} \quad (67)$$

$$F_{n_4,n_5,n_6}(X, Y, Z) = \sum_{n_3=0}^{\infty} P_{n_3,n_4,n_5,n_6}(X,Y) Z^{n_3} \quad (68)$$

$$F_{n_5,n_6}(X,Y,Z,R) = \sum_{n_4=0}^{\infty} P_{n_4,n_5,n_6}(X,Y,Z) R^{n_4} \quad (69)$$

$$F_{n_6}(X,Y,Z,R,S) = \sum_{n_5=0}^{\infty} P_{n_5,n_6}(X,Y,Z,R) S^{n_5} \quad (70)$$

$$F(X,Y,Z,R,S,T) = \sum_{n_6=0}^{\infty} P_{n_6}(X,Y,Z,R,S)\, T^{n_6} \quad (71)$$

Now, on taking equal to zero one by one and then taking two of them pair wise, three of them at a time, four of them at a time, five of them at a time and all of them; we get 63 equations. Now proceeding on the lines of Maggu and Singh T.P. et.al. and following the standard technique, which after manipulation gives the final reduced result as –

F(X,Y,Z,R,S,T)=

$$\frac{\mu_1\left(1-\frac{Z}{X}\right)F(Y,Z,R,S,T) + \mu_2\left(1-\frac{Z}{Y}\right)F(X,Z,R,S,T) + \mu_3\left(1-\frac{R}{Z}p_{34}-\frac{S}{Z}p_{35}-\frac{T}{Z}p_{36}\right)F(X,Y,R,S,T) + \mu_4\left(1-\frac{1}{R}\right)F(X,Y,Z,S,T) + \mu_5\left(1-\frac{1}{S}\right)F(X,Y,Z,R,T) + \mu_6\left(1-\frac{1}{T}\right)F(X,Y,Z,R,S)}{\lambda_1(1-X) + \lambda_2(1-Y) + \mu_1\left(1-\frac{Z}{X}\right) + \mu_2\left(1-\frac{Z}{Y}\right) + \mu_3\left(1-\frac{R}{Z}p_{34}-\frac{S}{Z}p_{35}-\frac{T}{Z}p_{36}\right) + \mu_4\left(1-\frac{1}{R}\right) + \mu_5\left(1-\frac{1}{S}\right) + \mu_6\left(1-\frac{1}{T}\right)}$$

(72)

For convenience, let us denote

$F(Y,Z,R,S,T) = F_1$

$F(X,Z,R,S,T) = F_2$

$F(X,Y,R,S,T) = F_3$

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F(X,Y,Z,S,T) = F_4

F(X,Y,Z,R,T) = F_5

F(X,Y,Z,R,S) = F_6

Also F(1,1,1,1,1,1) = 1, being the total probability.

On taking X=1 as Y,Z,R,S,T →1 , F(X,Y,Z,R,S,T) is of $\frac{0}{0}$ indeterminate form.

Now, on differentiating numerator and denominator of (72) separately w.r,t X, we have

$$1 = \frac{\mu_1 F_1}{-\lambda_1+\mu_1}$$

$$\Rightarrow F_1 = 1 - \frac{\lambda_1}{\mu_1}$$

Again differentiating (72) w.r.t Y at Y = 1

$$1 = \frac{\mu_2 F_2}{-\lambda_2+\mu_2}$$

$$\Rightarrow F_2 = 1 - \frac{\lambda_2}{\mu_2}$$

Again differentiating (72) w.r.t Z at Z = 1

$$1 = \frac{-\mu_1 F_1 - \mu_2 F_2 + \mu_3(p_{34}+p_{35}+p_{36})F_3}{-\mu_1 - \mu_2 + \mu_3(p_{34}+p_{35}+p_{36})}$$

$\Rightarrow F_3 = 1 - \frac{\lambda_1+\lambda_2}{\mu_3}$ [where $p_{34} + p_{35} + p_{36} = 1$]

Again differentiating (72) w.r.t R at R = 1

$$1 = \frac{\mu_3(-p_{34)}F_3 + \mu_4 F_4}{-\mu_3 p_{34} + \mu_4}$$

$$\Rightarrow F_4 = 1 - \frac{(\lambda_1+\lambda_{2)}p_{34}}{\mu_4}$$

Again differentiating (72) w.r.t S at S = 1

$$1 = \frac{\mu_3(-p_{35)}F_3 + \mu_5 F_5}{-\mu_3 p_{35} + \mu_5}$$

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$\Rightarrow F_5 = 1 - \frac{(\lambda_1+\lambda_2)p_{35}}{\mu_5}$

Again differentiating (72) w.r.t T at T = 1

$1 = \frac{\mu_3(-p_{36})F_3+\mu_6 F_6}{-\mu_3 p_{36}+\mu_6}$

$\Rightarrow F_6 = 1 - \frac{(\lambda_1+\lambda_2)p_{36}}{\mu_6}$

Algorithm

The following algorithm provides the procedure to determine the various queue characteristics of above discussed queue model:

Step1.Obtain the number of customers $n_1, n_2, n_3, n_4, n_5, n_6$.

Step2. Obtain the values of mean service rate $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6$.

Step3.Obtain the values of mean arrival rate λ_1, λ_2.

Step4.Obtain the values of the probabilities p_{34}, p_{35}, p_{36}.

Step5. Calculate the value of

(i) $F_1 = 1 - \frac{\lambda_1}{\mu_1}$

(ii) $F_2 = 1 - \frac{\lambda_2}{\mu_2}$

(iii) $F_3 = 1 - \frac{\lambda_1+\lambda_2}{\mu_3}$

(iv) $F_4 = 1 - \frac{(\lambda_1+\lambda_2)p_{34}}{\mu_4}$

(v) $F_5 = 1 - \frac{(\lambda_1+\lambda_2)p_{35}}{\mu_5}$

(vi) $F_6 = 1 - \frac{(\lambda_1+\lambda_2)p_{36}}{\mu_6}$

Step 6.Calculate

(i) $\rho_1 = 1 - F_1$

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(ii) ρ_2=1−F_2

(iii) ρ_3=1−F_3

(iv) ρ_4=1−F_4

(v) ρ_5=1−F_5

(vi) ρ_6=1−F_6

Step 7.Check $\rho_1, \rho_2, \rho_3, \rho_4, \rho_5, \rho_6$<1

If so, then go to step (8) else steady state condition does not holds good.

Step8. Calculate average no. of customers (Mean Queue Length)

$$L=\frac{\rho_1}{(1-\rho_1)}+\frac{\rho_2}{(1-\rho_2)}+\frac{\rho_3}{(1-\rho_3)}+\frac{\rho_4}{(1-\rho_4)}+\frac{\rho_5}{(1-\rho_5)}+\frac{\rho_6}{(1-\rho_6)}$$

Step9. Calculate variance of queue

$$V=\frac{\rho_1}{(1-\rho_1)^2}+\frac{\rho_2}{(1-\rho_2)^2}+\frac{\rho_3}{(1-\rho_3)^2}\frac{\rho_4}{(1-\rho_4)^2}+\frac{\rho_5}{(1-\rho_5)^2}+\frac{\rho_6}{(1-\rho_6)^2}$$

Step10.Calculate average waiting time for customers

$$E(W) = \frac{L}{\lambda}.$$

Mean queue length for the system

Average number of the customer (L)

$$= \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty}\sum_{n_6=0}^{\infty}(n_1+n_2+n_3+n_4+n_5+n_6)\,P_{n_1,n_2,n_3,n_4,n_5,n_6}$$

$$= \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty}\sum_{n_6=0}^{\infty} n_1\,P_{n_1,n_2,n_3,n_4,n_5,n_6} + \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty}\sum_{n_6=0}^{\infty} n_2\,P_{n_1,n_2,n_3,n_4,n_5,n_6} + \text{-------------------} + \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty}\sum_{n_6=0}^{\infty} n_6\,P_{n_1,n_2,n_3,n_4,n_5,n_6}$$

$$L = L_1 + L_2 + L_3 + L_4 + L_5 + L_6$$

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Where

$$L_1 = \frac{\rho_1}{(1-\rho_1)}, L_2 = \frac{\rho_2}{(1-\rho_2)}, L_3 = \frac{\rho_3}{(1-\rho_3)}, L_4 = \frac{\rho_4}{(1-\rho_4)}, L_5 = \frac{\rho_5}{(1-\rho_5)}, L_6 = \frac{\rho_6}{(1-\rho_6)}$$

After substituting the values, we get

$$\frac{\lambda_1}{\mu_1-\lambda_1} + \frac{\lambda_2}{\mu_2-\lambda_2} + \frac{\lambda_1+\lambda_2}{\mu_3-(\lambda_{1+\lambda_2})} + \frac{(\lambda_1+\lambda_2)p_{34}}{\mu_4-(\lambda_{1+\lambda_2})p_{34}} + \frac{(\lambda_1+\lambda_2)p_{35}}{\mu_5-(\lambda_{1+\lambda_2})p_{35}} + \frac{(\lambda_1+\lambda_2)p_{36}}{\mu_6-(\lambda_{1+\lambda_2})p_{36}}$$

Variance of Queue

$$V(n_1 + n_2 + n_3 + n_4 + n_5 + n_6) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty}\sum_{n_6=0}^{\infty}(n_1 + n_2 + n_3 + n_4 + n_5 + n_6)^2 P_{n_1,n_2,n_3,n_4,n_5,n_6} - L^2$$

$$= \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty}\sum_{n_6=0}^{\infty}(n_1)^2 P_{n_1,n_2,n_3,n_4,n_5,n_6} + \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty}\sum_{n_6=0}^{\infty}(n_2)^2 P_{n_1,n_2,n_3,n_4,n_5,n_6} + \text{----------------} + \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty}\sum_{n_6=0}^{\infty}(n_6)^2 P_{n_1,n_2,n_3,n_4,n_5,n_6} + 2\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty}\sum_{n_6=0}^{\infty}(n_1 n_2) P_{n_1,n_2,n_3,n_4,n_5,n_6} + \text{-----------} + 2\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty}\sum_{n_4=0}^{\infty}\sum_{n_5=0}^{\infty}\sum_{n_6=0}^{\infty}(n_5 n_6) P_{n_1,n_2,n_3,n_4,n_5,n_6} - L^2$$

$$V = \frac{\rho_1}{(1-\rho_1)^2} + \frac{\rho_2}{(1-\rho_2)^2} + \frac{\rho_3}{(1-\rho_3)^2} + \frac{\rho_4}{(1-\rho_4)^2} + \frac{\rho_5}{(1-\rho_5)^2} + \frac{\rho_6}{(1-\rho_6)^2}$$

After substituting values, we get

$$\frac{\lambda_1\mu_1}{[\mu_1-\lambda_1]^2} + \frac{\lambda_2\mu_2}{[\mu_2-\lambda_2]^2} + \frac{(\lambda_1+\lambda_2)\mu_3}{[\mu_3-(\lambda_1+\lambda_2)]^2} + \frac{(\lambda_1+\lambda_2)\mu_4 p_{34}}{[\mu_4-(\lambda_1+\lambda_2)\mu_4 p_{34}]^2} + \frac{(\lambda_1+\lambda_2)\mu_5 p_{35}}{[\mu_5-(\lambda_{1+}\lambda_2)\mu_5 p_{35}]^2} + \frac{(\lambda_1+\lambda_2)\mu_6 p_{36}}{[\mu_6-(\lambda_1+\lambda_2)\mu_6 p_{36}]^2}$$

Average waiting time for customers in the system

$$E(W) = \frac{L}{\lambda}$$

Numerical illustration

Give customers coming to three servers out of which one server consists two parallel channels and other consist of three parallel service channels and further these two service channels are linked with common server. The number of customers, mean service rate, mean arrival rate and associated probabilities are given as follows:

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S.No	**No. of Customers**	**Mean Service Rate**	**Mean Arrival Rate**	**Probabilities**
1	$n_1 = 5$	$\mu_1 = 4$	$\lambda_1 = 3$	$p_{34} = 0.4$
2	$n_2 = 8$	$\mu_2 = 5$	$\lambda_2 = 4$	$p_{35} = 0.3$
3	$n_3 = 6$	$\mu_3 = 8$		$p_{36} = 0.3$
4	$n_4 = 3$	$\mu_4 = 6$		
5	$n_5 = 4$	$\mu_5 = 3$		
6	$n_6 = 20$	$\mu_6 = 9$		

Find the mean queue length, variance of queue and average waiting time for customer.

Solution:-We have

$$\rho_1 = \frac{\lambda_1}{\mu_1} = \frac{3}{4} = 0.75$$

$$\rho_2 = \frac{\lambda_2}{\mu_2} = \frac{4}{5} = 0.8$$

$$\rho_3 = \frac{\lambda_1 + \lambda_2}{\mu_3} = \frac{7}{8} = 0.875$$

$$\rho_4 = \frac{(\lambda_1 + \lambda_2)p_{34}}{\mu_4} = \frac{7(0.4)}{6} = 0.467$$

$$\rho_5 = \frac{(\lambda_1 + \lambda_2)p_{35}}{\mu_5} = \frac{7(0.3)}{3} = 0.7$$

$$\rho_6 = \frac{(\lambda_1 + \lambda_2)p_{36}}{\mu_6} = \frac{7(0.3)}{9} = 0.23$$

The Mean queue length (Average no. of customers)

$$L = \frac{\rho_1}{(1-\rho_1)} + \frac{\rho_2}{(1-\rho_2)} + \frac{\rho_3}{(1-\rho_3)} + \frac{\rho_4}{(1-\rho_4)} + \frac{\rho_5}{(1-\rho_5)} + \frac{\rho_6}{(1-\rho_6)}$$

$$= 17.47$$

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Variance of queue

$$V=\frac{\rho_1}{(1-\rho_1)^2}+\frac{\rho_2}{(1-\rho_2)^2}+\frac{\rho_3}{(1-\rho_3)^2}\frac{\rho_4}{(1-\rho_4)^2}+\frac{\rho_5}{(1-\rho_5)^2}+\frac{\rho_6}{(1-\rho_6)^2}$$

$= 97.81$

Average waiting time for customer

$$E(W) = \frac{L}{\lambda} = \frac{17.47}{7} = 2.49$$

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