THINK INDIA JOURNAL

ISSN:0971-1260
Vol-22- Issue-14-December-2019

Weighted Similarity Measure Using Pseudo-metric in Neutrosophic Decision Making Environment

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***Abstract*—**.In this paper, we have proposed the normalized orthogonal distance between two neutrosophic sets and its properties. The similarity measure and weighted similarity measures are defined using the normalized orthogonal distance and the key role of weighted similarity measures in decision making in the neutrosophic environment is derived. Finally the application of normalized similarity measure in multi attribute decision making is illustrated through an example.
***Keywords*—** *Neutrosophic point, Normalized orthogonal distance, Similarity measure, Weighted similarity measure, Ideal attribute, Decision making.*

I. INTRODUCTION

The Oxford English dictionary defines the term fuzzy as not clear or vague. In 1965, Lotfi A Zadeh introduced the concept of mathematical representation of vague concept or imprecise boundaries as a fuzzy set in which each element have truth membership value in $[0,1]$. Fuzzy set became the effective tool to handle the vague concepts in day to day life problems. A generalization of fuzzy set theory, intuitionistic fuzzy was proposed by Attanassov in 1986 in which each element is associated with degree of member ship and non-membership values. As a generalization of fuzzy set and intuitionistic fuzzy set, neutrosophic set was defined with three different type of membership values by Smaranandache in 1995. In real world the practical problems are relat ed with incomplete, indeterminate and inconsistent information. Neutrosophic set is a powerful tool and most appropriate frame work for dealing incomplete, indeterminate and inconsistent information. The main objective of this paper is study the physical properties of normalised orthogonal distance and similarity relation in nuetrosophic environment. Finally investigate the procedure and key role of neutrosophic distances in multi attribute decision making situations.

II. PRELIMINARIES

Definition 2.1. [1] Let X be a non-empty set. A metric on a set X is a function $d : X \times X \to R$ satisfies the following axioms

1. $d(x,y) \geq 0$
2. $d(x,y) = 0 \Leftrightarrow x = y$
3. $d(x,y) = d(y,x)$
4. $d(x,y) \leq d(x,z) + d(z,x), \forall x,y,z \in X$

Definition 2.2 [2]. [1] Let X be a non-empty set. A Pseudo metric on a set X is a function $d : X \times X \to R$ satisfies the following axioms

1. $d(x,y) \geq 0$
2. $d(x,x) = 0$ [Possible $d(x,y) = 0$ if $x \neq y$]
3. $d(x,y) = d(y,x)$
4. $d(x,y) \leq d(x,z) + d(z,x), \forall x,y,z \in X$

Definition 2.3 [6] A neutrosophic set A on the universal set X is defined as $A = \{(x, t_A(x), i_A(x), f_A(x)) : x \in X\}$

ISSN:0971-1260
Vol-22- Issue-14-December-2019

where $t_A, i_A, f_A : X \to (^-0,1^+)$. The three components t_A, i_A and f_A represent membership value (Percentage of truth), indeterminacy (Percentage of indeterminacy) and non-membership value (Percentage of falsity) respectively. These components are functions of nonstandard unit interval $(^- 0,1^+)$.

Definition 2.4 [6]: Let X be a universe of discourse. A single valued neutrosophic set (SVNS) A over X is an object having the form $A = \{(x, t_A(x), i_A(x), f_A(x)) : x \in X\}$ where $t_A, i_A, f_A : X \to [0,1]$. The numbers $t_A(x), i_A(x)$ and $f_A(x)$ denote the degree of truth membership, indeterminacy membership and falsity membership of x in X respectively.

Definition 2.5. [6] Let A and B be two neutrosophic sets on X. Then A is contained in B, denoted as $A \subseteq B$ if and only if $A(x) \le B(x), \forall x \in X$, this means that $t_A(x) \le t_B(x), i_A(x) \le i_B(x)$ and $f_A(x) \ge f_B(x)$.

Definition 2.6. [6] The compliment of a neutosophic set $A = \{(x, t_A(x), i_A(x), f_A(x)) : x \in X\}$ is defined as

$A^C = \{(x, f_A(x), 1 - i_A(x), t_A(x)) : x \in X\}$.

Definition2.6. [3,6] Let A and B be two neutrosophic sets on X. The union C of A and B is denoted by C=A $\cup$B and defined as

$$C(x) = A(x) \vee B(x) = \{x, t_C(x), i_C(x), f_C(x)\}$$

where $t_C(x) = t_A(x) \vee t_B(x)$, $i_C(x) = i_A(x) \vee i_B(x)$ and $f_C(x) = f_A(x) \wedge f_B(x)$.

Let A and B be two neutrosophic sets on X. The intersection C of A and B is denoted by C=A$\cap$B and defined as

$$C(x) = \{x, t_C(x), i_C(x), f_C(x)\}$$

where $t_C(x) = t_A(x) \wedge t_B(x)$, $i_C(x) = i_A(x) \wedge i_B(x)$ and $f_C(x) = f_A(x) \vee f_B(x)$.

Definition 2.7. [3] Let A and B be neutrosophic sets on X. Then their sum A+B is a neutrosophic set on X, defined as follows

$$t_{A+B}(x) = \vee\{t_A(y) \wedge t_B(z) : x = y + z, y, z \in X\}$$
$$i_{A+B}(x) = \vee\{i_A(y) \wedge i_B(z) : x = y + z, y, z \in X\}$$
$$t_{A+B}(x) = \vee\{t_A(y) \wedge t_B(z) : x = y + z, y, z \in X\}$$

III. NORMALISED ORTHOGONOL DISTANCE

A neutrosophic set A means, each element x of the universal set X can be characterized by three components consisting of truth, indeterminacy and falsity membership values of x in X. In other words it is a function which maps the element x in X to triplet membership values $\{t_A(x), i_A(x), f_A(x)\}$. Geometrically it can be represented using three axis representation where each axis represents one subset. X axis represents truth membership values, Y-axis represents indeterminacy membership values and Z- axis represent falsity membership values. The range of variation of each axis is restricted to [0, 1] for SVNS. Considering the interval, an imaginary unit cube can be constructed and any point in X can be located in and on the unit cube **[2].**

Definition 3.1. Any objects in a neutrosophic set A defined on the universal set X can be represented as a vector in which the components t_A, i_A and f_A are marked on Cartesian system of rectangular coordinate system. It is denoted in vector form as follows

$$\vec{A}(x) = t_A(x)i + i_A(x)j + f_A(x)k$$

and its modulus, the distance between origin and the neutrosophic component of A is denoted and defined as

$$|A(x)| = \sqrt{(t_A(x))^2 + (i_A(x))^2 + (f_A(x))^2}$$

Definition 3.2. Let $A = \sum_{i=1}^{n} \frac{\langle t_A(x_i), i_A(x_i), f_A(x_i)\rangle}{x_i}$ and $B = \sum_{i=1}^{n} \frac{\langle t_B(x_i), i_B(x_i), f_B(x_i)\rangle}{x_i}$ be two neutosophic sets defined on a universal set $X = \{x_1, x_2, ..., x_n\}$ where the components of A are not a scalar multiple of B at each point x_i. The normalized orthogonal distance between two neutrosophic sets A and B can be denoted and defined as

ISSN:0971-1260
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$$d^{\perp}(A,B)=\sum_{i=1}^{n}\frac{\sqrt{(T_{AB}(x_i))^2+(I_{AB}(x_i))^2+(F_{AB}(x_i))^2}}{\max(|A(x_i)|,|B(x_i)|)}$$

where $T_{AB}(x_i)=[t_A(x_i)i_B(x_i)-i_A(x_i)t_B(x_i)], I_{AB}(x_i)=[i_A(x_i)f_B(x_i)-f_A(x_i)i_B(x_i)]$ and $F_{AB}(x_i)=[f_A(x_i)t_B(x_i)-t_A(x_i)f_B(x_i)]$

Example 3.1. Assume that $X=\{x_1,x_2\}$. Let A and B are two neutrosophic sets defined on the universal set X where $A=\frac{\langle 0.5,0.3,0.1\rangle}{x_1}+\frac{\langle 0.8,0.1,0.1\rangle}{x_2}$ and $B=\frac{\langle 0.7,0.2,0.1\rangle}{x_1}+\frac{\langle 0.6,0.3,0.1\rangle}{x_2}$. Then the normalized orthogonal distance between A and B is $d^{\perp}(A,B)=0.6000+0.12248=0.72248$

Definition 3.3. The normalized orthogonal distance $d^{\perp}(A,B)$ between two neutrosophic sets A and B (not collinear) which is defined on the universal set X satisfies the following axioms

1. $d^{\perp}(A,B)\geq 0$
2. $A=B\Leftrightarrow d^{\perp}(A,B)=0$
3. $d^{\perp}(A,B)=d^{\perp}(B,A)$
4. $d^{\perp}(A,C)\leq d^{\perp}(A,B)+d^{\perp}(B,C)$ where C is any third neutrosophic set.

Theorem 3.1. The distance $d^{\perp}(A,B)$ is a metric.

Definition 3.4. The normalized orthogonal distance between A and B hold the inequality $0\leq d^{\perp}(A,B)\leq 3n$ where n is the cardinality of universal set X.

IV. RESULTS AND DISCUSSION

A similarity measure or similarity function is a real-valued function that quantifies the similarity between two objects. Similarity measure take large values on similar objects and either zero or a negative value for very dissimilar objects. The concept of similarity is fundamentally important in almost every scientific field. The knowledge about similarity measure is necessary for data mining, pattern recognition, artificial intelligence and multi-agent system fields. In this section the similarity measure between two neutrosophic sets is defined using the normalized orthogonal distance. Similarity measures are inversely proportional to distance between the sets.

Definition 4.1. [7] A similarity measure between two neutrosophic sets A and B is a function defined as $S:X\times X\rightarrow[0,1]$ which satisfies the following properties

1. $S(A,B)\in[0,1]$
2. $S(A,B)=1\Leftrightarrow A=B$
3. $S(A,B)=S(B,A)$
4. $A\subset B\subset C\Rightarrow S(A,C)\leq S(A,B)\wedge S(B,C)$

Theorem 4.1. The function $S^{\perp}(A,B)=\frac{1}{1+d^{\perp}(A,B)}$ defined between two neutrosophic sets A and B using normalized orthogonal distance is a similarity measure.

Proof. To prove the function $S^{\perp}(A,B)=\frac{1}{1+d^{\perp}(A,B)}$ is a similarity measure, it is enough to prove that the function $S^{\perp}(A,B)$ satisfies the properties of definition 4.1.

Property 1. It is clear from the definition

Property 2. From the definition, $d^{\perp}(A,B)=0\Leftrightarrow A=B$

Therefore $S^{\perp}(A,B)=1\Leftrightarrow A=B$ where A and B are two non collinear neutosophic set.

ISSN:0971-1260
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Property 3. We know $d^{\perp}(A,B)=d^{\perp}(B,A)$, then by definition, $S^{\perp}(A,B)=S^{\perp}(B,A)$.

Property 4. Given $A\subset B\subset C$, then

$t_A(x)\le t_B(x)\le t_C(x), i_A(x)\le i_B(x)\le i_C(x)$ and $f_A(x)\ge f_B(x)\ge f_C(x)$. Then,

$i_C(x)\ge i_B(x)\Rightarrow t_A(x)i_C(x)\ge t_A(x)i_B(x).....(1)$

$t_C(x)\ge t_B(x)\Rightarrow t_C(x)i_A(x)\ge t_B(x)i_A(x).....(2)$

$(1)-(2), T_{AC}(x)\ge T_{AB}(x)$ .Similarly $I_{AC}(x)\ge I_{AB}(x)$.

Now consider $t_B(x)\le t_C(x)$

$\Rightarrow f_A(x)t_B(x)\le f_A(x)t_C(x)....(3)$ and

$f_B(x)\ge f_A(x)\Rightarrow f_B(x)t_B(x)\ge f_C(x)t_A(x)\Rightarrow -f_B(x)t_B(x)\le -f_C(x)t_A(x).....(4)$

$(3)+(4), F_{AB}\le F_{AC}$. Similarly $T_{Bc}\le T_{AC}$, $I_{Bc}\le I_{AC}$ and $F_{BC}\le F_{AC}$. Then

$d^{\perp}(A,B)\le d^{\perp}(A,C)\Rightarrow S^{\perp}(A,B)\ge S^{\perp}(A,C)$ and $d^{\perp}(B,C)\le d^{\perp}(A,C)\Rightarrow S^{\perp}(B,C)\ge S^{\perp}(A,C)$

$\Rightarrow S^{\perp}(A,C)\le S^{\perp}(A,B)\wedge S^{\perp}(B,C)$

Hence $S^{\perp}(A,B)$ is a similarity measure.

Definition 4.2. . Let $A=\sum_{i=1}^{n}\frac{\langle t_A(x_i), i_A(x_i), f_A(x_i)\rangle}{x_i}$ and $B=\sum_{i=1}^{n}\frac{\langle t_B(x_i), i_B(x_i), f_B(x_i)\rangle}{x_i}$ be two neutosophic sets defined on a universal set $X=\{x_1,x_2,...,x_n\}$. Let $w_i\in[0,1]$ be the weight of each element $x_i\,(i=1,2,...,n)$ with the property that $\sum_{i=1}^{n}w_i=1$.The weighted similarity measure using normalized orthogonal distance can be denoted and defined as follows

$$WS^{\perp}(A,B)=\frac{1}{1+\sum_{i=1}^{n}w_i\frac{\sqrt{(T_{AB}(x_i))^2+(I_{AB}(x_i))^2+(F_{AB}(x_i))^2}}{\max(|A(x_{i,})|,|B(x_i)|)}},$$

Definition 4.3. The weighted similarity measure using normalized orthogonal distance satisfies the following properties

1. $0\le WS^{\perp}(A,B)\le 1$
2. $WS^{\perp}(A,B)=WS^{\perp}(B,A)$
3. $WS^{\perp}(A,B)=1$ if $A=B$

V. APPLICATION OF NORMALISED SIMILARITY MEASURE IN DECISION-MAKING

A multi criteria decision - making technique using similarity measure in neutrosophic data is proposed here. Let $A=\{A_1,A_2,...,A_m\}$ be a set of m attributes and $C=\{C_1,C_2,...,C_n\}$ be a set of n criteria. Let w_j be the weight of the criterion C_j where $j=1,2,...,n$ fixed by decision maker such that each $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$. The characteristics of the attributes A_i where $i=1,2,...,m$ on criteria C_j where $j=1,2,...,n$ is denoted by the following SVNS form on C_j

$$\{\langle C_j, t_{A_i}(C_j), i_{A_i}(C_j), f_{A_i}(C_j)\rangle\}$$

ISSN:0971-1260
Vol-22- Issue-14-December-2019

A decision maker $D=(\alpha_{ij})_{m\times n}=\lfloor a_{ij},b_{ij},c_{ij}\rfloor$ where $i=1,2,...,m$ and $j=1,2,...,n$ can be constructed from the above equation which is the evaluation of an alternative A_i with respect to a criterion C_j by the expert or decision maker. In multi-criteria decision making neutrosophic environment, the concept of ideal point has been used to identify the best attribute in the decision set.

Definition 5.1. [7] In the decision making procedure, criteria are classified into two, according to their nature:
1. Benefit criteria: - Maximum operator is used for identifying ideal alternative in benefit criteria
2. Cost criteria:-Minimum operator is used for identifying ideal alternative in cost criteria.

Definition 5.2. [7] In the multi attribute decision making process, the ideal attribute α_j can be denoted and defined as follows for benefit criteria C_j.

$$\alpha_j=\left\langle \max_i a_{ij}, \min_i b_{ij}, \min_i c_{ij}\right\rangle=(a_j,b_j,c_j)$$

and for a cost criterion

$$\alpha_j=\left\langle \min_i a_{ij}, \max_i b_{ij}, \max_i c_{ij}\right\rangle=(a_j,b_j,c_j)$$

where $j=1,2,...,n.$

Definition 5.3. The similarity measure between an attribute A_i and the ideal attribute α_j can be defined as

$$WS^{\perp}(A_i,\alpha_j)=\frac{1}{1+\sum_{j=1}^{n} w_j d^{\perp}(A_i,\alpha_j)}$$

$$d^{\perp}(A_i,B)=\sum_{i=1}^{n}\frac{\sqrt{(T_{A_i\alpha_j})^2+(I_{A_i\alpha_j})^2+(F_{A_i\alpha_j})^2}}{\max(|A_i|,|\alpha_j|)}$$

$T_{A_i\alpha_j}=a_{ij}b_j-b_{ij}a_j$, $I_{A\alpha_j}=b_{ij}c_j-c_{ij}b_j$ and $F_{A\alpha_{j_i}}=c_{ij}a_j-a_{ij}c_j$, $|A_i|=\sqrt{(a_{ij})^2+(b_{ij})^2+(c_{ij})^2}$ and $|\alpha_j|=\sqrt{(a_j)^2+(b_j)^2+(c_j)^2}$ where $j=1,2,...,n$ and $i=1,2,...,m.$

Hence the similarity measure between each attribute and ideal attribute is calculated and the ranking order of all attributes can be determined using the relation

$$\sum_{j=1}^{n} WS^{\perp}(A_i,C_j)\ .$$

Then the best attribute can be selected easily.

VI. ALGORITHM FOR DECISION MAKING USING WEIGHTED SIMILARITY MEASURE IN NEUTROSOPHIC ENVIORNMENT

- Step 1. Develop a model for the decision.
- Step 2. Identify properly the attributes A_i where i=1,2,...,m and criteria where j=1,2,..,n in neutrosophic decision making
- environment.
- Step 3. Determine the weight $w_j\,(j=1,2,...,n)$ of each criterion
- Step 4. Construct a decision matrix $D=(\alpha_{ij})_{m\times n}$
- Step 5. Calculate the ideal attribute α_j using the evaluation of each attribute A_i on each criterion C_j

ISSN:0971-1260
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- Step 6. Calculate weighted similarity measure $WS^{\perp}(A_i, \alpha_j)$
- Step 7. Determine ranking order of all attributes using the calculation of $A_i^* = \sum_{j=1}^{n} WS^{\perp}(A_i, \alpha_j)$ corresponding to each
- criterion
- Step 8. Select the best attribute easily using the ranking order.

VII. CONCLUSION

The proposed normalized orthogonal distance and weighted similarity measure are one of the most generalized notions of classical theories to explain vague or uncertainty or indeterminacy environment. The decision making process using weighted similarity measure can be extended to different fields like engineering, medicine or highly complex decision making situations. The procedure proposed in this paper for decision making is convenient and simple to adopt for practical purposes

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