THINK INDIA JOURNAL

ISSN:0971-1260
Vol-22- Issue-14-December-2019

Using Mathematical Modeling For New Insights In Sustainable Development

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ABSTRACT

Natural Environment and its various ingredients like wild life, forest and other resources have attracted significant academic attention for the past several decades. All these ingredients not only provide stability to the ecosystems but also have significant economic value to the communities' dependent upon them. Proper utilization of these natural resources will help to maintain the economic growth, provide livelihood opportunities to the people and help to sustain such resources. Hence the study of wild life species, evolution of their populations and interrelationships between them has been crucially important. It has largely been an interdisciplinary subject which has attracted the attention of researchers from wide ranging subjects like physics, biological sciences, mathematicians, geographers etc.The issues have gained prominence in light of the Sustainable Development Goals (SDG) launched by United Nations.

Mathematical applications in ecology are important keeping in view of their predictive power in the most complex ecological dynamics. This research article puts the issues of mathematical modelling in to perspective of Sustainable Development and explores its utility for planning for Sustainable Development.

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BACKGROUND

The Sustainable Development Goals (SDG) of United Nations has attracted the attention of academic community across the world. The fast changes occurring in the environment and its repercussions for social and economic lives of the communities is now a major cause of concern. It is a major challenge to predict the environmental changes and its impact on living beings. For instance, the wild life experts find it difficult to do census for wild life species, estimate the population projections for future and thereby monitor the evolution of such species. Therefore the subject of environmental planning has become a prominent area of investigation for researchers. It will require a great deal of precision in our understanding of the natural world and our relationship with it. To acquire that understanding, progress in the mathematical sciences plays a big role in social, environmental, and economic sustainability of world. Therefore, mathematical applications in several areas of human enquiry have now attracted a great deal of academic interest. The mathematicians have shown concern over several real life problems which have often been too difficult to be handled through conventional experimentations. Results have been encouraging. Mathematical sciences have now come to the forefront of the developmental pursuits across the world. These mathematical interventions have been in diverse areas like epidemiology, wild life management etc. For a long time, there is an increasing concern to build human environment models for sustainable development (Philip, 2010; Singh, 2013).Though such concerns have been a major motivating factor for a long time, there is a renewed impetus in view of Sustainable Development Goals (SDG) of United Nations. Many developmental challenges could be solved if we can comprehend the environmental changes through the language of mathematics.

Comprehending Nature through Mathematics: Possibilities and Constraints

There has been an eternal quest in mankind to comprehend the nature, predict the outcomes of natural phenomenon and understand the natural events through the language of mathematics. Its predictive potential makes it a viable tool to forecast the outcomes of an observed phenomenon. Natural phenomenon includes several variable and parameters which makes them extremely complex. Mathematics can simplify observed phenomenon, develops a computationally viable description, depicts the patterns of interactions between the factors and thus identify the dominant driving factors which drive the evolution of the system. It is quite often said that mathematics is the language which nature follows (Marianne, 2018). The famous quote of Galileo makes it amply clear.

“Philosophy [nature] is written in that great book whichever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth.”

― Galileo Galilei, in IlSaggiatore (1623)

The role of mathematics to comprehend the physical world is well recognized and understood (Dhar 2018). There have been creative interventions by mathematicians to resolve some of the biggest mysteries of our Universe. For a long time the mathematicians had indulged only in the research problems related to physical sciences. It has not only helped the mathematicians to resolve the research problems of physics but have also utilized the opportunity to devise new tools of mathematics.

The role of mathematics is now seriously being taken in several other branches of human enquiry. Though mathematical application in biology & medicine has been examined by several scientists like Euler, Descartes, Helmholtz, Volterra & Von Neuman, such efforts had been limited. However, twentieth century has witnessed a spurt of activities in innovative application of mathematical tools in Ecology and Environmental Sciences. Since the beginning of 20th century the famous "*Lotka Volterra Model*" attracted the attention of mathematicians and subsequently it was realized that potential of mathematics need to sufficiently explored in relatively less explored field.

Ecological dynamics now carries a lot of significance for the mathematicians. It presents fascinating situations for the mathematicians to intervene and develop predictive mathematical models to comprehend the evolution of ecosystems. It has further created new areas of research which has also prompted mathematicians to explore new mathematical tools. There is hardly any field of biosciences where mathematicians have not made any perceptible impact. The mathematical techniques which are used to explore biosciences are Classical Mathematics Techniques, Probabilistic and Statistical Techniques, Operation Research Techniques and Modern Mathematical Techniques etc.

However mathematical modeling has its own disadvantages also. It depicts a simplified version of the reality. Therefore its predictions are likely to deviate from observed reality. However sometimes model is the only viable option available to investigate the phenomenon. A model can be viable to the extent it is solvable analytically. The models are especially useful in those circumstances where it is impossible to perform experiments. For example evolution of wild study of after effects of nuclear blasts, study of propagation of pollution in atmosphere, possible effects of drug interactions in the human body, disaster evacuation strategy development, performing unethical experiments on humans etc

PURPOSE OF MODELING

The models has been used in several different contexts of scientific enquiry (Walker 1963 and May 1976).The word Model is used to refer to different notions needed for scientific communications. As per the Webster's dictionary, the Model can be used to as synonym for Example, Pattern, Exemplar, Paradigm, Ideal etc. It further states that Model helps to visualize something which cannot be observed directly. Several nomenclatures like cortical Model,

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Conceptual Model, Verbal Model, Postulation Model, Geometrical Model, Mathematical Model, Material Model etc have quite often been cited in research literature and such terms have been utilized as per the context of research questions. Walker (1963) says that Models are required to make predictions and not to interpret the process described by equation. However such a statement gives an erroneous conclusion that aim of science is just to predict and not to understand the underlying processes. The view point of Walker (1963) has been criticized for making a misleading statement about purpose of science just as the predictor of future. Scientific method is expected to provide overall understanding of nature and not just to predict the future. An interesting reference to ancient Babylonian astronomers can help to understand this point. They were able to predict the motions of star within remarkable degrees of accuracy without properly understanding the underlying mechanisms. However Ionic philosophers crude model of Universe ultimately unfolded the new paradigms of 20th century physics. However the observed facts in natural state are not the ultimate. It is quite possible that such observed facts represent only the visible part of the whole. Models can help us to generalize such observations. Poincare (1950) has concluded that without generalizations predictions are impossible.

The Model Building

Though there is no rigid set of step by step process, the Model Building has to pass through certain broad steps like Collection of Prior Knowledge of the Modeling Problem, Simplification of the Problem, Mathematical Formulation, Analysis and Validation (IIT Web Site). The Modeling Process has to be very much clear about the goal for the modeling. The goal setting requires a thorough understanding of the prior knowledge available for the Modeling Problem. The researcher has to find out the data from real world. Once all the background knowledge and data has been collected the problem can be simplified. Only certain dominant driving force of the system can be taken in to account. Subsequently the mathematical formulation and analysis of the model can be done and lastly validation of the model from real world needs to be done (IITK Web Site-Lecture Notes).However the model building is an evolutionary process. With the more and more creative ways of mathematical formulation we can reach finer levels describing the system dynamics.

Mathematical Applications for Ecology

The mathematical applications are important primarily due to its cost effectiveness. It is difficult and often very costly proposition to carry out trustworthy experimental investigations in ecological settings, epidemiology or other similar areas. Mathematics comes to the rescue of the policy planners under such circumstances. The mathematical applications initially became popular with the ground breaking work of Ross(2011).The underlying complexity in mathematical comprehension of such issues was generally considered to be challenging because very few researchers have both the mastery of mathematical tools and the necessary epidemiological knowledge for its application. However even the great mathematicians who have ventured in to this challenging area started it as a beginners. However there has been a

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contention among such beginners that main laws which regulate epidemics are as simple as the law of gravitation, though the application of them to the diverse conditions which govern epidemics is often of considerable complexity (Anderson,1982).Therefore it was felt as early as beginning of 20th century that a reasoned analysis of the relations between various driving factors of a disease or wild life population to predict the evolution of various factors which influence it. The study carried out by Ross (1911) in connection with official report(as cited by Waite,1910) on the " Prevention of Malaria in Mauritius " is worth citing .The study created tremendous enthusiasm among the researchers and triggered a great deal of further research(Lotka,1923; Lotka,1912; Waite,1910).

Mathematical modeling in ecology has not developed in isolation. The applications of mathematical applications in ecology has also stimulated the interests of mathematicians to make the mathematics interesting for their students through real life examples. Several authors have reviewed such mathematical applications from time to time Hayslett, 1983).Mathematical modeling of wild life evolution can take in to account the specific parameters of the species' environment, survival rates and reproduction and try to evolve a mathematical framework to predict population growth. Some assumptions are necessary to make the framework computable. However there should not be any compromise with the dominant patterns and factors which are driving evolution. Such decisions have to be taken while we develop the framework.

It started playing important role for developing management strategies and formulation of policies (Conway, 1977).Initially the mathematical interventions in environmental issues have been of academic interest only, it very soon caught the imagination of environmental planners. The modeling has found to be useful to understand whether a specific species is going to be endangered (University of Southern Denmark, 2019).However it depends primarily upon the quality of information we are feeding to deduce the results. Estimation of risk of extinction will depends on how precise is our calculation of key driving factors of our model as the environment changes (Fernando and Conde, 2019).Such wild life modeling can have very practical implications to take timely management actions to prevent extinction of endangered species. During the last century the extinction rates of the species have enhanced 1000 times the background rates and the number of species which have threatened due to climate change have multiplied many times(Barnosky et al 2011).The climate change has direct impact on food availability , survival rates, birth rates, frailty, genetic characters etc.The variation in these parameters can affect extinction rates of the species(Pearson e al 2014).These emerging concerns about climate change crisis demands that population dynamics should be understood in an even more precise way which could help to predict extinction of species (Femando et. al.2018)

This article traces the genesis of using mathematics for the environmental conservation, highlights its actual utilization and future prospects.

ECOLOGICAL MODELLING

Ecology is primarily a dynamic study which looks in to the systematic interactions in the groups of organisms and with the biotic and abiotic factors constituting their environment. The study of wild life populations is crucial for developmental planning. There are several attributes of animals which may be quite distinct from their collective or statistical behaviour as a group.

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Therefore group behaviour of wild life species is of significance. The populations of various wild life species living in any natural environment can well be compared with the tissues /organs constituting various organisms. They are functional units which interact among themselves and also with biotic as well as abiotic factors of natural environment. The degree of such interactions is vitally important for the sustenance of such communities of wild life species and also for the habitat they live in.

Such interaction exhibits themselves in the fascinating patterns of wild life populations. Ecological researchers have always tried to examine underlying factors responsible for such patterns. It has been the central theme of the Mathematical Ecology. There are several reviews on the issues in population modelling, including effects of age and stage structure, environmental and demographic stochasticity, and the interplay between evolution and ecology (Fryxell, 2016 as cited on Oxford Bibliographies).

Mathematical Ecologists have developed several approaches through which evolution of populations can be comprehended. The key elements of the Mathematical Ecology are as follows (Fryxell, 2016).

1. To measure and characterize different patterns of population fluctuation,
2. To develop mathematical frameworks based on the key drivers of processes and mechanisms which might have caused specific patterns of population fluctuation,
3. To develop ways to test such models using real life observations.

Thus the mathematical models predict how populations change in size and composition over time. The fluctuations in the population are primarily dependent upon factors lie Natality, Mortality, Emigration & Immigration. There are several additional concerns like Compensatory Mortality, Additive Mortality, Density Dependence, Carrying Capacity etc. The initial population model was very simple. This model depicted population evolution without any environmental constraint .In other words it showed population variation in an ideal environment with unlimited resources. There is unlimited food available to species and therefore nothing can check their growth. Variation turns out to be a J-Shaped curve as given below. However the real world has finite resources and therefore the species cannot grow in unlimited way. Its growth is constrained by available resources for the species. Once the resources reach the limit of carrying capacity the population tend to fluctuate around it.If there is situation, such that none of the constraints are present to disturb the growth and proliferation of an organism, then the organism will keep on multiplying. Such a growth pattern of an organism is known as exponential growth. Nearly all populations will tend to grow exponentially as long as the resources are available.

MATHEMATICS: A TOOL FOR SDGs

Environment and Anthroposhere have a great degree of impact on each other. Hans-Joachim Schellnhuber in 1998 postulated the coupled and dynamic relationship between the environment and humans. He further emphasized that a dynamic interrelation between Human Beings and Environment is vital for the Sustainable Development. It is now widely believed that knowledge about the interaction between Man and Nature is the crux of the Sustainable Development. This knowledge need to be captured, documented and utilized for strategic planning for Sustainable

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Development. A Mathematical Model involving the key concepts and parameters of coupled dynamics of environment and humans can help to depict co evolution between environment and humans (Philips 2010).

Mathematics as a tool of Knowledge Generation can be immensely useful in the context of Sustainable Development (UNDP, 2017). SDGs are well integrated framework of 17 goals and 169 targets which are intimately linked to each other. Due to their integrated nature, they can be put within a Mathematical Model. Further, SDGs are time bound goals with specifically defined outcomes .Hence there is an urgent need to project the quantifiable outcomes.

Therefore pursuits of SDGs require predictive capacity in planning process. However some of the SDGs cannot be predicted through any kind of experimentation in real life due to political, financial or logistic reasons. We have to perform their predictions under simulated conditions. Mathematical modeling is the only answer under such circumstances. There is an urgent need to find out the factors which might influence these targets /goals and the underlying mechanisms which drive such a change. Therefore there is an urgent need for mathematical interventions in the field of SDGs.

Such initiatives have not been sufficiently explored in the context of SDGs so far. We have enormous possibilities to mathematically model the expected outcomes of our interventions for SDGs. Mathematical modeling tries to create a simplified picture of a real world phenomenon, take in to account the essential interdependent driving forces of the phenomenon and project the outcomes theoretically. In the context of Sustainable Development Goals the SDG inter linkages are the essential interdependencies to model the outcomes (UNDP, 2017).The results of these models can be immensely useful to explore policy options. The predictive power of mathematical models will help the planners to take timely interventions and thereby achieve the goals in the given time frame.

There has already been a huge pool of such mathematical frameworks within which researchers have tried to anticipate ecological change. Such initiatives if utilized in the framework of SDGs will be immensely useful.

Modeling Man Nature Interactions

The Environment consists of high levels of orderliness and systematic patterns of wild life populations which have fascinated human beings. Such predictability in the environmental processes have attracted the attention of mathematicians and interactions between Man and Environment have been a subject matter of intensive intellectual activity in mathematics. Mathematical applications in environment can be traced back to ancient civilizations as an attempt to understand and analyze the world around them.

The researchers have been inquisitive about the interaction between population dynamics and

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environment. It has been a field of active research and has been well documented (Peterson, 1972 and Cohen, 1995). Ever since Thomas Malthus wrote his pioneering *Essay on the Principle of Population* this field has stimulated the interest of researchers from a wide range of scientific disciplines.

The Malthus had assumed that population tends to grow exponentially whereas food production grows linearly. The mismatch between the population and food availability leads to natural checks (e.g. famines etc) in population growth. It was first mathematical description of growth of population with natural resources which has been extensively cited in research literature. The research carried out by Malthus was followed up by several authors. The logistic equation developed by ecologist and mathematician Pierre-François Verhulsttakes in to account the intrinsic growth rate and carrying capacity (Verhulst, 1838 as cited in Wade ,2016).Inspired by the logistic equation, Alfred Lotka,in 1910, proposed a pair of equations, to describe autocatalytic chemical reactions (Lotka, 1910 as cited in Wade ,2016).It further paved the way for well-known predator–prey Lotka–Volterra model forming the basis for Ecological Modelling(Wade 2016). The "*LotkaVolterra Model*" which ingeniously described the Prey Predator interactions inspired research mathematical ecology to describe several environmental issues. The researchers have included several new mathematical factors in this model to further enrich it.Pearl and Reed used it to predict population growth in the United States (Pearl and Reed, 1920 as cited in Wade,2016). Jacques Monod further examined the logistic equation and selected bacteria for the purpose, due to the doubling-time of these organisms being far shorter than the larger life forms originally the subject of the model (Monod, 1942 , as cited in Wade ,2016).U.S. National Academy of Sciences, in 1963 published *The Growth of World Population* which deliberated about consequences of global population growth. The growth rate of human population stimulated concern around that time as it had reached its peak annual rate of two percent. In the year 1968, Paul Ehrlich published *The Population Bomb* which drew the attention on the issue of population growth, food production, and the environment. The population growth model was difficult to be conceived initially primarily the limited capacities of existing mathematical frameworks, techniques and capabilities for computer based calculation.

The advent for computer simulation techniques have opened up tremendous scope for mathematical modelling primarily due to capacities to incorporate complex interaction of Man and Nature in Mathematical frameworks and carry out mathematical calculations. However the human capacity to model the Man Nature interactions took a fillip with mathematical computation capabilities. The Club of Rome had released its World Model in 1972, which was the first initiative to model the population growth based on computer. It predicted that global carrying capacity could be reached within 100 years. The efforts to include the environmental changes like global trends of energy consumption, carbon dioxide emissions, nitrogen deposition, deforestation etc and its consequential effect on Population Growth has been subject matter of subsequent initiatives to predict the population growth. However the efforts to

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mathematical model the Man Nature interactions have been difficult and have often been criticized for oversimplification of a complex reality. Such mathematical initiatives have been blamed for being poor fit to actual realities which they aim to simulate and often making wrong predictions.

However the mathematical depictions of population-environment interactions have by and large progressed with growing human capabilities for computation. The field despite its criticism has slowly matured as researchers have gradually started understanding nuances of the complex relationships of Man and Nature through increasing computational capabilities. The demographers, geographers, anthropologists, economists, and environmental scientists have immensely contributed to comprehend such complex dynamics of Man and Nature. The impact of temporal and spatial changes in population patterns of species, changes in food availability for the species, environmental changes and their complex interrelationships have been successfully comprehended through mathematical modelling. A host of new tools (geographic information systems, remote sensing, computer-based models, and statistical packages) and with evolving theories on human-environment interactions have been handy for such Mathematical Modelling

Rising International Concerns for Mathematical Modelling

The complex interrelationships between demographic trends and environment have been a subject matter of research interests for a long time. The environment is the source of the primary food production in the nature and therefore it has strong bearing upon the interplay of animal population along the food chain. The environmental changes have led to drastic changes in food production and natural habitats of wildlife species which has given rise to the changes in the interactional patterns among the various trophic levels of the food chain. The researchers working in the field of Mathematical Ecology have tried to bring such changes in to a mathematical framework, calculate the possible long term changes in populations of wild life species and thereby predict the changes in environment. The mathematical techniques have therefore proved to be useful. Sustainable Development Goals (SDG) of United Nations has raised several challenges for development planners particularly in developing countries where the policy planners are not adequately capacitated to plan the strategies and monitor the outcomes. As SDGs are time bound, their effective implementation will requires predictive methods. The Mathematical Tools can prove handy to adequately plan the developmental interventions and project the outcomes. There has been a growing realization that Mathematical Modelling can be a viable tool for Sustainable Development Planning.

Hence there is a need for the mathematicians to work on interdisciplinary areas of Environment, identify the driving forces for Environmental Change and predict its future evolution. The predictive powers of Ecological Models have been increasing with the powerful computing methods developed during past few decades. New possibilities have therefore arisen and moreover new expectations have also come up keeping in view of Sustainable Development Goals. There have been several research studies which underlines the role of mathematics in the study of Ecology and examine the relevance of Mathematical Modelling tools in the context of Sustainable Development Goals of United Nations .All such studies have concluded that Mathematical Modelling has tremendous potential to strengthen policy making and there is a need to inform Sustainable Development Policies through deepening of modelling" skills" of" government" officials" and" academics"

As per the information given on the web site of United Nations, the interdependence and complementarity of slower population growth with poverty eradication, economic development, and the adoption of sustainable consumption and production patterns have been highlighted in Programme of Action of the International Conference on Population and Development (United Nations web site). It further highlights the trends in the world population with the sustainable development of urban and rural areas, the eradication of poverty, hunger and preventable diseases.

The mathematical depiction of the evolution of animal species in natural environment has attracted the attention of mathematicians for a long time. Mathematical Modelling is basically an exercise of simulation. It provides an opportunity to explore the ideas which cannot be pursued through laboratory studies due to logistics, political or financial reasons. Further it helps to identify driving forces for evolution of the ecosystems. It also examines and helps to find data needs and knowledge gaps to understand ecological dynamics (Jackson et al, 2000). It is increasingly being realized that Mathematical formulation of environmental issues can help people understand more on environmental losses than mere awareness.

The worldwide concerns for the Sustainable Development have intensified interest to utilize mathematics as a tool to comprehend the ecological processes, predict their evolution and utilize this knowledge to address the concerns of Sustainable Development (Cabezas&Fath 2002; Grafton &Technique 1997). Fast computers and graphical software package have opened new possibilities for predictive mathematical modelling (Jackson et al 2000).Philosophy of Mathematical Modelling and current approaches for mathematical modelling of ecosystems has been extensively reviewed (Evans, 2012).

Mathematical Modelling of ecological dynamics can be quite useful to enhance the predictive power for ecological evolution (Evans, 2012).However it’s a challenging job due to complex interplay of large number of parameters involved. We need to identify the dominant factors which drive the evolution, understand the underlying mechanisms and thereby predict the evolution. It’s an interdisciplinary exercise and requires innovative application of mathematical tools. There have been imaginative efforts to evolve mathematical frameworks which can help to comprehend complex processes of ecological evolution. Most of such frameworks have tried to simplify the underlying mechanisms to an extent that application of mathematical tools is possible.

Twentieth century has witnessed a spurt of activities in mathematical formulation of ecological processes(Renert,na).There have been several paradigmatically different approaches to mathematically understand the natural processes (Baker, 2011) e.g. we have deterministic versus stochastic, Mathematical versus statistical models and Continuous versus discrete time models (Cain, 2014).However despite several ingenious efforts there has been a lack of consensus and conclusive frameworks to comprehend the evolution of animal populations. Moreover several studies have shown serious reservation about the validity of such Mathematical Models as realistic descriptors of ecological dynamics (Hall 1988). There is need to revisit the development of this vital subject. The following issues are the on the forefront of the research in mathematical ecology

a) The role of Models and usefulness of mathematical models in study of ecological systems
b) Approaches for Model Building
c) Critical appraisal of status of Mathematical approaches

CONCLUSIONS

Pursuits for Sustainable Development Goals require a better understanding of ecological dynamics and enhanced predictive capacity of ecological evolution. It will help us to take timely measures to mitigate the undesirable impact in ecological dynamics. This paper has given rise to a perspective on the Mathematical Modeling for the Developmental issues. Prey Predator Modeling which has attracted the attention of the researchers across the world has been briefly reviewed and emerging multifarious dimensions of Prey Predator Modeling have been highlighted. Starting with the simple Lotka Volterra Model in the beginning of 20th Century this interesting model has been interpreted from several points of views. Most of these models have satisfactorily predicted the patterns of evolution. The usefulness of mathematical models will ultimately depend upon how reliably such models can include environmental changes .It will require better understanding of the processes of the ecological dynamics and skilful incorporation of such processes inthe mathematical frameworks. Hence mathematical modeling requires innovative application of mathematical tools and frameworks in ecosystems to project their evolution in future. We feel that there is an increasing need to further strengthen research work in this vital area and utilize the full potential of mathematics for predicting the ecological processes.

With the increasing human capabilities for computing, Mathematics can prove to be a powerful tool to study ecological dynamics. It has got fresh impetus within the context of Sustainable Development. Some of the finest studies in Ecological Modelling have been done in developed countries. The research problems within the context of developing countries are yet to be meaningfully addressed. However the concerns are gradually picking up in developing countries. Ugwa&Agwu(2012) has emphasized that mathematical modelling can play a vital role for a sustainable development in Nigeria. However existing research literature proves that mathematical modelling as a tool holds a great promise for future. The recent initiative of the

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United Nation to hold a conference for policy makers to sensitize them about mathematical modelling and its use in policy planning is a welcome initiative.

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