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Sustainable Fuzzy Inventory Models With Incorporation Of Environmental Costs

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Abstract:

An economic production quantity inventory model is formulated with the inclusion of environmental costs in treatment and disposal of the waste generated from the various stages in the production process. Yager's ranking method for fuzzy numbers is utilized to find the inventory policy in terms of the fuzzy total cost. Finally a numerical example is given to illustrate the model.

Keywords: Waste disposal, Inventory, Fuzzy number, Yager's ranking.

1. Introduction:

Waste generation within the manufacturing process is steadily analyzed by the firms due to the regulatory pressures emphasized by the government agencies. The strict legislations and regulations influence the manufacturing firm's activities which in turn act as the main driver of firm's environmental activities. The manufacturing industries have started to practice environmental sustainable measures by implementing waste minimization techniques. Waste minimization is defined as the reduction of the generation of waste at source through the efficient use of raw materials, energy and water. The environmental costs are the costs associated with waste management which comprises of the costs of waste treatment and disposal. The environmental cost is added to the purchasing and holding costs of the inventory required for the continuous production without shortages. The environmental issues started surfacing and becoming a public concern Richter(1996) picked up the problem of schrady(1967) who had developed an EOQ model with instanteous production, repair rate and no disposal and discussed the same with disposal (Richter, 1996). In 2011, Huang Jing has given a complete definition of Environmental costs. In precise environmental costs of a firm relates to all costs incurred in its relation to environmental damage and protection.

In this paper, we convert the inventory model into fuzzy inventory model. The approach of this paper is to find the optimal order quantity with minimum cost determined from Yager's ranking method.

2. Preliminaries

Definition 2.1

Let X be a non empty set. A **fuzzy set** $\tilde{A}$ of X is defined as

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) / x \in X\}$$

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Where $\mu_{\tilde{A}}(x)$ is called the membership function which maps each element x in X to a real number in the interval [0,1].

Definition 2.2

The set of elements that belong to the fuzzy set $\tilde{A}$ atleast to the degree α is called the α level set or **α-cut**.

$$A(\alpha) = \{x \in X: \mu_{\tilde{A}}(x) \geq \alpha\}$$

Definition 2.3

A fuzzy set $\tilde{A}$ defined on the set of real numbers R, is said to be a **Fuzzy number** if its membership function $\mu_{\tilde{A}}: R \rightarrow [0,1]$ as the following characteristics,

(i) $\tilde{A}$ is fuzzy convex

(ii) $\tilde{A}$ is normal

(iii) $\tilde{A}$ is piecewise continuous

Definition 2.4

A fuzzy set $\tilde{A}$, membership function $\mu_{\tilde{A}}$ that maps each element in X to a real number in the interval [0,1]. A **trapezoidal fuzzy number** is represented by $\tilde{A} = \{a_1, a_2, a_3, a_4\}$, where a_1, a_2, a_3, a_4 are real numbers and its membership function is given below.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{for} \quad x < a_1 \\ \frac{x-a_1}{a_2-a_1} & \text{for} \quad a_1 \leq x \leq a_2 \\ 1 & \text{for} \quad a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & \text{for} \quad a_3 \leq x \leq a_4 \\ 0 & \text{for} \quad x > a_4 \end{cases}$$

2.5 Yager's Ranking Method

If the α cut of any fuzzy number $\tilde{A}$ is $[A_L(\alpha), A_R(\alpha)]$, then its ranking index is $\frac{1}{2}\int_0^1 [A_L(\alpha), A_R(\alpha)]\,d\alpha$.

3. Assumptions and Notations

The model is developed on the following assumptions and notations.

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3.1 Assumptions

1. Demand for items from inventory is continuous and at a constant rate.
2. Production rate is continuous; greater than demand rate and it runs to replenish inventory at regular intervals.
3. Production set up cost is fixed (independent of quantity produced)
4. The lead time is fixed.
5. The total allocator costs for the effluent treatment, solid waste treatment and gas emissions treatment comprises of the allocator costs of the treatment caused by the wastes of different kind in all the operations executed in various stages involved in the production process.
6. The processing cost comprises of all the associated costs of executing all the operations involved in the production process.

3.2 Notations

D	Demand per unit of time
P	Production per unit of time
X	$\frac{D}{P}$
M	1-x, the fraction of time the production process spends actually idling
A	fixed set up cost per production run
h	holding cost per unit of time
V	Processing costs for all the operations executed in various stages involved in the production process. Processing cost covers input costs, labor costs and energy consumption costs.
L	total cost of liquid effluent treatment
S	total cost of solid waste treatment
G	total cost for Gas treatment
x	total allocator costs of the effluents treatment caused by the waste in all the operations
y	total allocator costs of the solid treatment and waste disposal caused by the waste in all the operations
z	total allocator cost of gas emission treatment caused by the waste in all the operations

3.3 Model Formulation

Nowadays the manufacturing firms are facing severe pressures from government in terms of legislations and regulations which drives them to carry out environmental activities. The main motive of these activities is to prevent the environment from the effects of the disposal of waste of different maters. The formulation of an environmentally sustainable IM with the costs associated with the treatment and disposal of the waste generated at various stages of the production process.

The EPQ cost per unit of time $C(Q)=\frac{AD}{Q}+\frac{hQM}{2}$

The processing cost per cycle $C_V(Q)=V$

Waste treatment and disposal cost per cycle $C_{TD}(Q)=xL+yS+zG=F(say)$

Total cost per unit of time

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$$TC(\mathrm{Q}) = \mathrm{C(Q)} + \frac{\mathrm{C}_P(Q) + C_{TD}(Q)}{T}, where T = \frac{Q}{D}$$

$$TC(Q) = \frac{AD}{Q} + \frac{hQM}{2} + \frac{D}{Q}(V + F)$$

The objective is to find the optimal quantity. The necessary condition for minimum is $\frac{\partial TC(Q)}{\partial Q} = 0$.

The optimal solution is $Q = \sqrt{\frac{2D(A+V+F)}{hM}}$

4. Fuzzy inventory model

Let $\tilde{D}, \tilde{A}, \tilde{h}, \tilde{V}, \tilde{\mathrm{F}}, \tilde{\mathrm{M}}$ be trapezoidal fuzzy numbers and they are defined by using α-cuts as follows.

$$A(\alpha_A) = [\mathrm{L}_A^{-1}(\alpha_A), \mathrm{R}_A^{-1}(\alpha_A)]$$

$$\mathrm{D}(\alpha_D) = [\mathrm{L}_D^{-1}(\alpha_D), \mathrm{R}_D^{-1}(\alpha_D)]$$

$$\mathrm{V}(\alpha_V) = [\mathrm{L}_V^{-1}(\alpha_V), \mathrm{R}_V^{-1}(\alpha_V)]$$

$$\mathrm{F}(\alpha_F) = [\mathrm{L}_F^{-1}(\alpha_F), \mathrm{R}_F^{-1}(\alpha_F)]$$

$$\mathrm{h}(\alpha_h) = [\mathrm{L}_h^{-1}(\alpha_h), \mathrm{R}_h^{-1}(\alpha_h)]$$

$$\mathrm{M}(\alpha_M) = [\mathrm{L}_M^{-1}(\alpha_M), \mathrm{R}_M^{-1}(\alpha_M)]$$

$\mathrm{TC}(Q)$ can be rewritten as

$$\mathrm{TC}(Q) = \frac{\tilde{A}\tilde{D}}{Q} + \frac{\tilde{D}\tilde{V}}{Q} + \frac{\tilde{D}\tilde{F}}{Q} + \frac{\tilde{h}Q\tilde{M}}{2}$$

$$K_1(\alpha_D, \alpha_A) = \frac{1}{4}\left\{\int_0^1 \mathrm{L}_D^{-1}(\alpha_D)\,\mathrm{d}\alpha_D . \int_0^1 \mathrm{L}_A^{-1}(\alpha_A)\,\mathrm{d}\alpha_A + \int_0^1 R_D^{-1}(\alpha_D)\,\mathrm{d}\alpha_D . \int_0^1 R_A^{-1}(\alpha_A)\,\mathrm{d}\alpha_A\right\}$$

$$K_2(\alpha_D, \alpha_V) = \frac{1}{4}\left\{\int_0^1 \mathrm{L}_D^{-1}(\alpha_D)\,\mathrm{d}\alpha_D . \int_0^1 \mathrm{L}_V^{-1}(\alpha_V)\,\mathrm{d}\alpha_V + \int_0^1 R_D^{-1}(\alpha_D)\,\mathrm{d}\alpha_D . \int_0^1 R_V^{-1}(\alpha_V)\,\mathrm{d}\alpha_V\right\}$$

$$K_3(\alpha_D, \alpha_F) = \frac{1}{4}\left\{\int_0^1 \mathrm{L}_D^{-1}(\alpha_D)\,\mathrm{d}\alpha_D . \int_0^1 \mathrm{L}_F^{-1}(\alpha_F)\,\mathrm{d}\alpha_F + \int_0^1 R_D^{-1}(\alpha_D)\,\mathrm{d}\alpha_D . \int_0^1 R_F^{-1}(\alpha_F)\,\mathrm{d}\alpha_F\right\}$$

$$K_4(\alpha_h, \alpha_M) = \frac{1}{4}\left\{\int_0^1 \mathrm{L}_h^{-1}(\alpha_h)\,\mathrm{d}\alpha_h . \int_0^1 \mathrm{L}_M^{-1}(\alpha_M)\,\mathrm{d}\alpha_M + \int_0^1 R_h^{-1}(\alpha_h)\,\mathrm{d}\alpha_h . \int_0^1 R_M^{-1}(\alpha_M)\,\mathrm{d}\alpha_M\right\}$$

$$\mathrm{TC}(Q) = \frac{K_1(\alpha_D, \alpha_A)}{Q} + \frac{K_2(\alpha_D, \alpha_V)}{Q} + \frac{K_3(\alpha_D, \alpha_F)}{Q} + \frac{QK_4(\alpha_h, \alpha_M)}{2}$$

$$\frac{\partial TC(Q)}{\partial Q} = 0$$

$$\frac{\partial}{\partial Q}\left(\frac{K_1(\alpha_D, \alpha_A)}{Q} + \frac{K_2(\alpha_D, \alpha_V)}{Q} + \frac{K_3(\alpha_D, \alpha_F)}{Q} + \frac{QK_4(\alpha_h, \alpha_M)}{2}\right) = 0$$

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$$\frac{-1}{Q^2}[K_1(\alpha_D,\alpha_A)+K_2(\alpha_D,\alpha_V)+K_3(\alpha_D,\alpha_F)]+\frac{K_4(\alpha_h,\alpha_M)}{2}=0$$

$$Q^* = \sqrt{\frac{2[K_1(\alpha_D,\alpha_A)+K_2(\alpha_D,\alpha_V)+K_3(\alpha_D,\alpha_F)]}{K_4(\alpha_h,\alpha_M)}}$$

5. Numerical Example

Consider a Fuzzy inventory system with following data.

$$\tilde{D} = (49000, 49500, 50500, 51000)$$
$$\tilde{A} = (50, 75, 125, 150)$$
$$\tilde{V} = (867, 870, 876, 879)$$
$$\tilde{F} = (848, 850, 854, 856)$$
$$\tilde{h} = (3, 4, 6, 7)$$
$$\tilde{M} = (0.1333, 0.2333, 0.4333, 0.5333)$$

$$D(\alpha_D) = (49000+500\alpha, 51000-500\alpha)$$
$$\mathrm{A}(\alpha_A) = (50+25\alpha, 150-25\alpha)$$
$$\mathrm{V}(\alpha_V) = (867+3\alpha, 879-3\alpha)$$
$$\mathrm{F}(\alpha_F) = (848+2\alpha, 856-2\alpha)$$
$$\mathrm{h}(\alpha_h) = (3+\alpha, 7-\alpha)$$
$$\mathrm{M}(\alpha_M) = (0.1333+0.1\alpha, 0.5333-0.1\alpha)$$

$$K_1(\alpha_D,\alpha_A) = \frac{1}{4}\left\{\int_0^1 (49000+500\alpha)\,\mathrm{d}\,\alpha.\int_0^1 (50+25\alpha)\,\mathrm{d}\,\alpha + \int_0^1 (51000-500\alpha)\,\mathrm{d}\,\alpha.\int_0^1 (150-25\alpha)\,\mathrm{d}\,\alpha\right\}$$
$$K_1(\alpha_D,\alpha_A) = 2514062.5$$

$$K_2(\alpha_D,\alpha_V) = \frac{1}{4}\left\{\int_0^1 (49000+500\alpha)\,\mathrm{d}\,\alpha.\int_0^1 (867+3\alpha)\,\mathrm{d}\,\alpha + \int_0^1 (51000-500\alpha)\,\mathrm{d}\,\alpha.\int_0^1 (879-3\alpha)\,\mathrm{d}\,\alpha\right\}$$
$$K_2(\alpha_D,\alpha_V) = 21826687.5$$

$$K_3(\alpha_D,\alpha_F) = \frac{1}{4}\left\{\int_0^1 (49000+500\alpha)\,\mathrm{d}\,\alpha.\int_0^1 (848+2\alpha)\,\mathrm{d}\,\alpha + \int_0^1 (51000-500\alpha)\,\mathrm{d}\,\alpha.\int_0^1 (856-2\alpha)\,\mathrm{d}\,\alpha\right\}$$
$$K_3(\alpha_D,\alpha_F) = 21301125$$

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$$K_4(\alpha_h,\alpha_M)=\frac{1}{4}\left\{\int_0^1(3+\alpha)\,\mathrm{d}\alpha.\int_0^1(0.1333+0.1\alpha)\,\mathrm{d}\alpha+\int_0^1(7-\alpha)\,\mathrm{d}\alpha.\int_0^1(0.5333-0.1\alpha)\,\mathrm{d}\alpha\right\}$$

$$K_4(\alpha_h,\alpha_M)=0.94575$$

$$Q^*=\sqrt{\frac{2[K_1(\alpha_D,\alpha_A)+K_2(\alpha_D,\alpha_V)+K_3(\alpha_D,\alpha_F)]}{K_4(\alpha_h,\alpha_M)}}$$

$$Q^*=\sqrt{\frac{2[2514062.5+21826687.5+21301125]}{0.94575}}$$

$$Q^*=9824.45$$

5. Conclusion:

The main purpose of this paper is to study the integrated models under fuzzy environment. This fuzzy model assists in determining the optimal order quantity amidst the existing fluctuations. This model assists the production sectors in building greener environment.

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