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Congruence Relation On D-Algebras

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ABSTRACT: The notion of d-algebras introduced J.Neggers and H.S.Kim [7] which is generalization of BCK-algebras. They investigated several relations between d-algebras and BCK-algebras. Ideal theory in d-algebras introduced J.Neggers, Y.B.Jun and H.S.Kim [8] and investigated some relations. In this paper i introduced Congruence relation on d-algebras, proved ~ is equivalence relation and ~(0) is d^*- ideal.

Key words: BCK-algebra ,d-sub algebra ,d^*-ideal , equivalence relation.

1.PRELIMINARIES:

1.1 Definition[7]: A d-algebra is a non-empty set X with a constant 0 and a binary operator * satisfying the following axioms

(i) $x * x = 0$
(ii) $0 * x = 0$
(iii) $x * y = 0$ and $y * x = 0 \Rightarrow x = y$.

1.2 Example[7]:

Let X = {0,1,2} be a set with the following cayley table

$*$	0	1	2
0	0	0	0
1	2	0	2
2	1	1	0

Then (x,∗,0) is a d-algebra

1.3 Example[7]: 2 Let X = {0,a,b,c} be a set with the following cayley table

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	b
b	b	b	0	0
c	c	c	A	0

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Then $(X, *, 0)$ is a d-algebra.

1.4 Definition [8]: Let X be a d-algebra and I be a subset of X, then I is called d-ideal of X if it satisfies the following conditions

(i) $0 \in I$
(ii) $x * y \in I$ and $y \in I \Rightarrow x \in I$
(iii) $x \in I$ and $y \in X \Rightarrow x * y \in I$

1.5 Example [8]: Let X= {0,1,2,3} be a d-algebra in which the operation $*$ defined as follows

$*$	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	1	0

In X the sets $I_1 = \{0,1\}$ and $I_2 = \{0,2\}$ are d-ideals of X, while $I_3 = \{0,3\}$ and $I_4 = \{0,1.2\}$ are not d-ideals of $X = \{0,1,2,3\}$.

1.6 Definition[8]: Let $(X, *, 0)$ be d-algebra and $x \in X$ .Define $x * x = \{x * a / a \in X\}$. X is said to be edge d-algebra if for any $x \in X$, $x * X = \{x, o\}$.

1.7 Lemma[7]: Let $(X, *, 0)$ be an edge d-algebra ,then $x * 0 = x$ for any $x \in X$.

We assume that $(X, *, 0)$ is a d-algebra with edge property.

1.8 Lemma[7]: Let $(X, *, 0)$ be a d-algebra with edge property ,then $(x * y) * z = (x * z) * y$, for all $x,y,z \in X$

1.9 Proposition[7]: For any d-algebra X ,and for all $x,y,z \in X$, we have

(1) $x * (x * y) = y$, for all $x,y \in X$
(2) $x * (x * (x * y)) = x * y$
(3) $0 * (x * y) = (0 * x) * (0 * y)$
(4) $x * 0 = 0 \Rightarrow x = 0$
(5) $((x * z) * (y * z)) * (x * y) = 0$
(6) $x \leq y \Rightarrow x * z \leq y * z$ and $z * y \leq z * x$
(7) $((x * y) * (x * z)) * (z * y) = 0$
(8) $(x * y) = (x * z) \Rightarrow y = z$ (left cancellation law hold)
(9) $(x * y) * 0 = (x * 0) * (y * 0)$
(10) $x * (y * z) \geq (x * y) * z$
(11) $(x * z) * (y * z) = (x * y)$
(12) $(x * 0) * 0 = x$

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1.10 Definition : A d-ideal I of d-algebra X is said to $d^{\#}$-ideal if for any $x,y,z \in X$,

$x * z \in I$ whenever $x * y \in I$ and $y * z \in I$.

1.11 Definition: A $d^{\#}$-ideal I of a d-algebra X is called d*-ideal if $x * y \in I$ and $y * x \in I \Rightarrow$

$(x* z) * (y* z) \in I$ and $(z * x) * (z * y) \in I$, for all $x,y,z \in X$.

2. MAIN PART :

2.1 Definition: Let I be d*-ideal of a d- algebra X.

Define $x \sim y$ if and only if $x * y \in I$ and $y* x \in I$, for any $x,y \in X$.

2.2 Theorem: Let I be d*-ideal of d-algebra X. Then ~ is equivalence relation on X

Proof: Define ~ on X by $x \sim y$ if and only if $x* y \in I$ and $y * x \in I$, for any $x,y \in X$.

Claim: To show that ~ is congruence relation:

(i) Since $0 \in I \Rightarrow x * x = 0 \in I$

$\Rightarrow x \sim x$, for any $x \in I$.

Therefore ~ is reflexive.

(ii) Let $x, y, z \in X$

Suppose $x \sim y$

Since $x \sim y$ if and only if $x * y \in I$ and $y * x \in I$

$\Rightarrow y * x \in I$ and $x* y \in I$

$\Rightarrow y \sim x$.

Therefore ~ is symmetric.

(iii) Let $x, y, z \in X$.

Suppose $x \sim y$ and $y \sim z$

$\Rightarrow x * y$, $y* x \in I$ and $y * z \in I$ and $z * y \in I$

$\Rightarrow x* z$, $z * x \in I$ (By 1.10).

$\Rightarrow x \sim z$.

Therefore ~ is transitive.

Therefore ~ is equivalence relation on X.

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2.3 Definition: An equivalence relation ~ on d-algebra X is a congruence relation on d-algebra X satisfying $x * y \in I$, $y * x \in I \Rightarrow (x * y) * (y * z) \in I$ and $(z * x) * (z * y) \in I$, for all $x, y, z \in X$

2.4 Theorem: Suppose ~ is congruence relation. Then ~(0) is d*-ideal.

Proof: (i) Since $0 \sim 0 \Rightarrow 0*0$ and $0*0 = 0 \in \sim(0)$

Therefore $0 \in \sim(0)$.

(ii) Suppose $y * x \in \sim(0)$ and $x \in \sim(0)$.

Since $x \in \sim(0) \Rightarrow x \sim 0$ if and only if $x * 0$, $0 * x$.

Since $y * x \in \sim(0) \Rightarrow (y * x) \sim 0$ if and only if $(y * x) * 0$, $0 * (y * x)$

$\Rightarrow (y * 0) * (x * 0)$, $(0 * y) * (0 * x)$ (By 1.9).

$\Rightarrow (y * 0) * 0$, $(0 * y) * 0$ (By def 1.1)

$\Rightarrow (y * 0) * (0 * 0)$, $(0 * 0) * (y * 0)$ (By 1.9)

$\Rightarrow (y * 0)$, $(0 * y) \Rightarrow y \sim 0 \Rightarrow y \in \sim(0)$.

(iii) Suppose $x \in \sim(0)$ and $y \in X$

Since $x \in \sim(0)$ if and only if $x * 0$, $0 * x$.

Now $x * 0$, $0 * x = x * (0 * y)$, $(0 * y) * x$ (By 1.1)

$x * y = x * ((y * 0) * 0)$ (By 1.9)

$= (x * (y * 0)) * 0 = (x * 0) * (y * 0) * 0$ (By 1.9 (9))

$= (x * 0) * (0 * 0)$ (By 1.1)

$= (x * 0) * 0 = (x * 0)$.

$x * y = x * ((x * (x * y)$ (By 1.9)

$= (x * y) * (x * x)$

$= (x * 0) * 0$ (By 1.1)

$= (0 * y) * x = (0 * x)$.

$x * y = (x * 0)$ and $(0 * x) \Rightarrow x \in \sim(0)$.

Therefore $x * y \in \sim(0)$.

(iv) Suppose $x * y \in \sim(0)$ and $y * z \in \sim(0)$

Since $x * y \in \sim(0)$ if and only if $(x * y) * 0$, $0 * (x * y)$.

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Since $y * z \in \sim(0)$ if and only if $(y * z) * 0 , 0 * (y * z)$.

Now $x * z = x * ((z * 0) * 0) = (x * ((z * 0)) * (x * 0)$

$= (x * z) * (x * 0) * (x * 0)$

$= (x * z) * 0$ (By 1.1).

$\in \sim(0)$.

$x * z = x * (x * (x * z))$ (By 1.9)

$= (x * x) * (x * (x * z))$

$= 0 * ((x * x) *(x * z))$

$= 0 * (0 * (x * z)) = 0 * (x * z) \in \sim(0)$

Therefore $x * z \in \sim(0)$.

(v) Suppose $x * y \in \sim(0)$ and $y * x \in \sim(0)$.

Since $x * y \in \sim(0)$ if and only if $(x * y) * 0 , 0 * (x * y)$.

Since $y * x \in \sim(0)$ if and only if $(y * x) * 0 , 0 * (y * x)$.

Now $(x * z) * (y * z) = (x * y) * z$ (By 1.9)

$= (x * y) *(x * (x * z))$

$= (x * y) *(x * x)*(x * z))$

$= (x * y) * (0 * (x * z))$(By 1.1)

$= (x * y) * 0$ (By 1.1)

$\in \sim(0)$.

$(x * z) * (y * z) = (x * y) * z$ (By 1.9)

$= (x * y) *(x * (x * z))$

$= (x * z) *(x * (x * y))$ (By edge property)

$= (x * z) * ((x * x) * (x * y))$ (By 1.9)

$= (x * z) * (0 * (x * y))$ (By 1.1)

$= 0 * ((x * z) * (x * y))$ (By edge property)

$= 0 * (x * (z * y))$

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= (0 * x) * (0 * (z * y)) (By 1.9)

= (0 * x) * ((0 * z) * (0 * y)) (By 1.9)

= (0 * x) * (0 * (0 * y))

= 0 * ((0 * x) * (0*y))

= 0 * (0 * (x * y)) = 0 * (x * y) $\in$ ~(0) .

Therefore (x * z) * (y * z) $\in$ ~(0) .Therefore~(0) is d*-ideal.

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