# A Study on Inventory of Deteriorating Things Having Price Reliant Demand, Time Reliant Holding Cost.

**Balvinder Singh Sandhu**

Lovely Professional University, Phagwara, Punjab. (balvinder.singh@lpu.co.in).

## Abstract

A deterministic inventory model is developed in which demand is reliant on price; cost for holding inventory is reliant on time. In this study, a generalized policy is developed for deteriorated items in which deterioration rate and holding cost are function of time and demand rate is a depending upon selling price. Maximization of profit function approach has been adopted. Shortages can take place with backlog.

*Keywords: Inventory, Price-dependent demand, holding cost, deterioration, shortages.*

## 1. Introduction

Many factors affect the inventory systems. Out of these factors demand factors is very important and demand depends on a number of other factors. Out of these factors demand is very important. Further demand depends upon a number of parameters. In this theory author is taking demand which is price oriented. A.H. Lau, H.S. Lau[1] presented a pricing model inventory and presented different belongings of demand curve shape. Mukherjee et al.[2] gave the EOQ model on price dependent demand. Roy, A [6] has given detailed discussion about price oriented demand.

## 2. Assumptions and Notations

| | | |
|---|---|---|
| $I_1(t)$ | : | Inventory between the time 0 to $t_1$. |
| $I_2(t)$ | : | Inventory between the time $t_1$ to $t_2$. |
| $t_1$ | : | Time when there is no shortage. |
| $t_2$ | : | Length of each ordering cycle. |
| D(p) | : | Selling price (p) depending demand |
| $C_H(t)$ | : | Cost of holding per unit time is time dependent. |
| $\theta(t)$ | : | Time varying deterioration. |
| $C_U$ | : | Purchasing cost. |
| $C_S$ | : | Shortage cost. |
| $C_o$ | : | Ordering cost. |
| $I_{max}$ | : | Maximum inventory level. |
| Q | : | Order quantity (same for all cycles) |
| $I_d$ | : | Amount of stock loss due to deterioration. |
| p | : | Selling price . |

 

**Assumptions:**

1. Singe item inventory is considered.
2. Infinite planning horizon.
3. Replenishment size is finite.
4. Deterioration rate is stable.
5. Deteriorated units are not reparable and replaceable during the period.
6. $d(p) = r - sp; where; r > 0 and 0 < s < 1$
7. $C_H(t) = \alpha + \beta t, where; \alpha > 0, \beta > 0$
8. Shortages are totally backlogged.

## 3. Mathematical modeling of problem

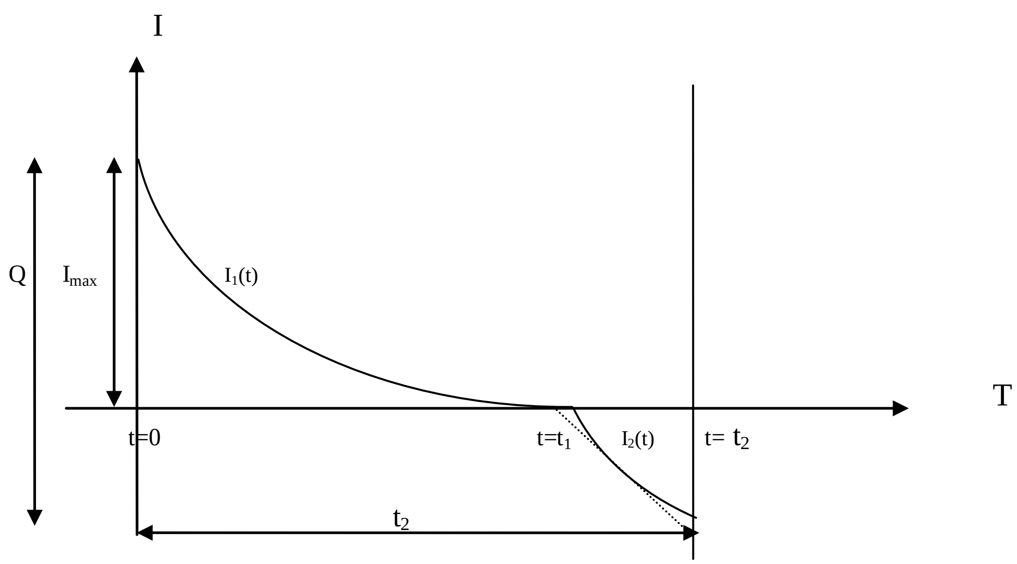


Figure-1: Graphical representation for the inventory system

From the above diagram

$$\frac{d}{dt} I_1(t) + \theta t I_1(t) = -(r - sp), \qquad 0 \le t \le t_1 \qquad (1)$$

and $$\frac{d}{dt} I_2(t) = -(r - sp), \qquad t_1 \le t \le t_2 \qquad (2)$$

Using $I_1(t_1) = 0$ and $I_1(0) = I_{max}$, solve equation (1)

 

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$$I_1(t)=(r-sp)\left[(t_1-t)+\theta\left(\frac{1}{6}t_1^3+\frac{1}{3}t^3-\frac{1}{2}t_1t^2\right)+\theta^2\left(\frac{1}{40}t_1^5-\frac{1}{15}t^5-\frac{1}{12}t_1^3t^2+\frac{1}{8}t_1t^4\right)\right] \quad (3)$$

and $I_{\max}=(r-sp)\left[t_1+\frac{\theta}{6}t_1^3+\frac{1}{40}\theta^2t_1^5\right]$ (4)

Solution of equation (2), using boundary condition $I_1(t_1)=0$ is

$$I_2(t)=-(r-sp)(t_1-t) \quad (5)$$

***Holding cost***

$$C_H=\int_0^{t_1}(\alpha+\beta t)I(t)dt$$

$$=(r-sp)\int_0^{t_1}(\alpha+\beta t)\left[(t_1-t)+\theta\left(\frac{1}{6}t_1^3+\frac{1}{3}t^3-\frac{1}{2}t_1t^2\right)+\theta^2\left(\frac{1}{40}t_1^5-\frac{1}{15}t^5-\frac{1}{12}t_1^3t^2+\frac{1}{8}t_1t^4\right)\right]dt$$

$$=(r-sp)\left[\alpha\left\{\frac{1}{2}t_1^2+\frac{1}{12}t_1^4+\frac{1}{90}\theta^2t_1^6\right\}+\beta\left\{\frac{1}{6}t_1^3+\frac{1}{40}t_1^5+\frac{1}{336}\theta^2t_1^7\right\}\right]$$

***Shortage cost***

$$C_S=-\int_{t_1}^{t_2}I_2(t)dt=-\int_{t_1}^{t_2}-(r-sp)(t_1-t)dt=\frac{1}{2}(r-sp)(t_2-t_1)^2$$

***Quantity of deteriorated stock per cycle***

$$I_d=I_{\max}-\text{Total demand during }[0,t_1]$$

$$=I_{\max}-\int_0^{t_1}(r-sp)dt$$

$$=(r-sp)\left[\frac{\theta}{6}t_1^3+\frac{1}{40}\theta^2t_1^5\right]$$

$$=(r-sp)\left[t_1+\frac{\theta}{6}t_1^3+\frac{1}{40}\theta^2t_1^5\right]-(r-sp)t_1$$

***Order Quantity***

Q= Amount of deteriorated inventory during the cycle +Total demand during the cycle $[0,\ t_2]$

 

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$$\Rightarrow Q = I_d + \int_0^{t_2} (r - sp)dt$$

$$= (r - sp)\left[\frac{\theta}{6}t_1^3 + \frac{1}{40}\theta^2 t_1^5\right] + (r - sp)t_2$$

$$= (r - sp)\left[\frac{\theta}{6}t_1^3 + \frac{1}{40}\theta^2 t_1^5 + t_2\right]$$

***The average overall profit per cycle***

TPF=P($t_2$, $t_1$,p) = Total sale-[Ordering cost + Inventory cost + Holding cost + Shortage cost]/ Length of ordering cycle.

$$= p(r - sp) - \frac{1}{t_2}\left[\begin{array}{l} C_o + C_U\,(r - sp)\left[\frac{\theta}{6}t_1^3 + \frac{1}{40}\theta^2 t_1^5 + t_2\right] + \\ (r - sp)\left[\alpha\left\{\frac{1}{2}t_1^2 + \frac{1}{12}t_1^4 + \frac{1}{90}\theta^2 t_1^6\right\} + \beta\left\{\frac{1}{6}t_1^3 + \frac{1}{40}t_1^5 + \frac{1}{336}\theta^2 t_1^7\right\}\right] \\ + C_S\frac{1}{2}(r - sp)(t_2 - t_1)^2 \end{array}\right] \tag{6}$$

Now P($t_2$, $t_1$,p) is a function of $t_2$, $t_1$,p. So, the conditions for the total profit to be maximized are

$$\frac{\partial P(t_2, t_1, p)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial P(t_2, t_1, p)}{\partial t_2} = 0$$

Satisfying,

$$\frac{\partial^2 P(t_2, t_1, p)}{\partial t_1^2} < 0 \quad , \quad \frac{\partial^2 P(t_2, t_1, p)}{\partial t_2^2} < 0 \text{ and}$$

$$\left(\frac{\partial^2 P(t_2, t_1, p)}{\partial t_1^2}\right)\left(\frac{\partial^2 P(t_2, t_1, p)}{\partial t_2^2}\right) - \left(\frac{\partial^2 P(t_2, t_1, p)}{\partial t_1 \partial t_2}\right)^2 > 0 \tag{7}$$

Now, $\frac{\partial P(t_2, t_1, p)}{\partial t_1} = 0$

$$\Rightarrow \begin{bmatrix} C_U\,(r-sp)\left[\frac{\theta}{2}t_1^{\,2}+\frac{1}{8}\theta^2 t_1^{\,4}\right]+ \\ (r-sp)\left[\alpha\left\{t_1+\frac{1}{3}t_1^{\,3}+\frac{1}{15}\theta^2 t_1^{\,5}\right\}+\beta\left\{\frac{1}{2}t_1^{\,2}+\frac{1}{8}t_1^{\,4}+\frac{1}{48}\theta^2 t_1^{\,6}\right\}\right]-C_S(r-sp)(t_2-t_1) \end{bmatrix}=0 \quad (8)$$

and $\dfrac{\partial P(t_2, t_1, p)}{\partial t_2}=0$ gives

$$\begin{bmatrix} t_2\{C_U\,(r-sp)+C_S(r-sp)(t_2-t_1)\}- \\ \left\{\begin{matrix} C_o+C_U\,(r-sp)\left[\frac{\theta}{6}t_1^{\,3}+\frac{1}{40}\theta^2 t_1^{\,5}+t_2\right]+ \\ (r-sp)\left[\alpha\left\{\frac{1}{2}t_1^{\,2}+\frac{1}{12}t_1^{\,4}+\frac{1}{90}\theta^2 t_1^{\,6}\right\}+\beta\left\{\frac{1}{6}t_1^{\,3}+\frac{1}{40}t_1^{\,5}+\frac{1}{336}\theta^2 t_1^{\,7}\right\}\right] \\ +C_S\frac{1}{2}(r-sp)(t_2-t_1)^2 \end{matrix}\right\} \end{bmatrix}=0 \quad (9)$$

Equations (8) and (9) can be solved with the help of computer software. The obtained values of $t_1$ and $t_2$ must satisfy equation (7) . After fulfilling the conditions (7) the best solutions will be obtained.

## 4. Numerical example

Let r=50, s=0.05, $\theta=0.01$, $C_U$=0.1, $C_S$=3, $C_o$=0.5, $\alpha$ =0.5, $\beta$ =0.9, p=20. Using these values of parameters, value of $t_1$=0.1593 and $t_2$ =0.1899

$$\left.\frac{\partial^2}{\partial t_1^{\,2}}P(t_2, t_1, p)\right]_{(t_1,t_2)} = -943.835<0 \quad , \quad \left.\frac{\partial^2}{\partial t_2^{\,2}}P(t_2, t_1, p)\right]_{(t_1,t_2)} = -774.051<0 \quad \text{and}$$

$$\left.\frac{\partial^2}{\partial t_1^{\,2}}P(t_2, t_1, p)\right]_{(t_1,t_2)} \times \left.\frac{\partial^2}{\partial t_2^{\,2}}P(t_2, t_1, p)\right]_{(t_1,t_2)} - \left(\left.\frac{\partial^2}{\partial t_1 \partial t_2}P(t_2, t_1, p)\right]_{(t_1,t_2)}\right)^2 = 131422.0>0$$

and then obtain optimal point $t_1=t_1^{*}=0.1593$ unit time and optimal length of ordering cycle $t_2=t_2^{*}=0.1899$unit time. Optimal maximum level $I^{*}_{max}$ =7.8387 and the optimal order quantity $Q^{*}$ = 9.3386 and optimal maximum average total profit per unit time $TPF^{*}$ = 970.304

 

## 5. Conclusion and Future research

This study is useful where deterioration takes place and cost of inventory holding depends upon time . This research may be extended to the model to allow for constant deterioration . Further for extension of research, time dependent demand, fixed holding cost etc. can taken into account.

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