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Some New Results on Certain Topological Indices of Signed Graphs

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Abstract: Topological indices of a graph are real numbers related to a graph which are structural invariant. There is vertex degree based topological indices. This paper extends the concept of topological indices of graph to signed graphs. We compute hyper Zagreb indices, forgotten topological indices, reduced second Zagreb indices of signed graph and first and second Zagreb coindices. This paper also computes the topological indices of signed graphs based on the net degree of the graph and elucidate the formulas relating these topological indices of signed graph to its underlying graph based on both degree and net degree of vertices.

Keywords: Positive and negative hyper-Zagreb indices, positive and negative forgotten topological indices, positive and negative Zagreb coindices, net degree.

Subject Classification: MSC 2010: 05C22, 05C07, 05C75

Introduction

For basic concepts of graph theory which are not defined in this paper, we refer to [2,3,12,20]. Moreover, for the concepts of signed graph theory, see [21,22,11]. Unless otherwise mentioned, all graphs considered here are simple, finite, connected and undirected.

For a graph, $G = (V, E)$, V represents a finite non empty set of objects called vertices and E denotes the twoelement subset of the vertex set Vcalled edges. The number of edges incident to a vertex v is called the degree of that vertex in G, denoted by $d_G(v)$. For simplicity, we use d_i instead of $d_G(v_i)$.

A $signed\ graph$, denoted by $S(G,\sigma)$ is a graph in which each edge is given a sign that is either positive or negative (see [21]). The $signature$ or $sign\ function$ of S, $\sigma: E(G) \rightarrow \{+,-\}$ assigns the signs, either $+$ or $-$, to every edge in G. It is not defined on half edges and they are positive on free loops. The unsigned graph G is called the $underlying\ graph$ of the signed graph S.

In a signed graph S, an edge is $positive$ or $negative$ in accordance with its signature $\sigma(e)$ is positive or negative. The $positive\ degree$, $d_S^+(v)$, of a vertex v is the number of edges with a positive sign incident on it. Similarly, the $negative\ degree$ of v, $d_S^-(v)$, is the number of negative edges incident on v. Then, $d_S(v) = d_S^+(v) + d_S^-(v)$.

A $topological\ index$ is a real number of a graph that determines the topology and is a structural invariant [13]. In other words, a topological index is a numerical parameter that is

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structural invariant under isomorphism. Many topological indices have been defined and studied extensively so far. The well-known topological indices are defined in terms of degree, distance, eccentricity, colour etc. taken separately or together. This paper considers the topological indices that are defined with respect to the degree of the vertices of the graph.

The $first\ and\ second\ Zagreb\ indices$, denoted by $M_1(G)$and $M_2(G)$ is defined in [8] as:

$$M_1(G) = \sum_{v_i \in V(G)} d_i^2,$$

$$M_2(G) = \sum_{v_i v_j \in E(G)} d_i d_j.$$

These notions can be extended to signed graphs as follows:

Let S be a signed graph and let d_i^+ and d_i^- be the positive and negative degree of the vertex v_i in S. Then, the $first\ positive\ Zagreb\ index$ of S, denoted by $M_1^+(S)$, is defined in [18] as,

$$M_1^+(S) = \sum_{v_i \in V(G)} (d_i^+)^2,$$

the $first\ negative\ Zagreb$ index of S is denoted by $M_1^-(S)$ and is defined in [18] as,

$$M_1^-(S) = \sum_{v_i \in V(G)} (d_i^-)^2,$$

and the $first\ mixed\ Zagreb\ index$ of S is denoted by $M_1^*(S)$ and is defined in [18] as,

$$M_1^*(S) = \sum_{v_i \in V(G)} d_i^+ d_i^-.$$

The $second\ Zagreb\ index$ for a signed graph S is defined in a similar way in [18] as follows:

Let S be a signed graph and let d_i^+ and d_i^- be the positive and negative degree of the vertex v_i in S.Then, the $second\ positive\ Zagreb\ index$ of S is denoted by $M_2^+(S)$ is defined as,

$$M_2^+(S) = \sum_{v_i v_j \in E(G)} d_i^+ d_j^+;\ \ 1 \le i \ne j \le n$$

the $second\ negative\ Zagreb\ index$ of S is denoted by $M_2^-(S)$ is defined in [18] as,

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$$M_2^-(S) = \sum_{v_i v_j \in E(G)} d_i^- d_j^- ; \;\; 1 \le i \ne j \; \le n$$

and the $second\ mixed\ Zagreb\ index$ of S is denoted by $M_2^*(S)$ and is defined in [18] as,

$$M_2^*(S) = \sum_{v_i v_j \in E(G)} d_i^+ d_j^- ; \;\; 1 \le i \ne j \; \le n.$$

I. TOPOLOGICAL INDICES OF SIGNED GRAPHS BASED ON DEGREE OF VERTICES

A. Hyper Zagreb Indices of Signed Graphs

The $hyper\ Zagreb\ index$ was introduced in [19] and is defined as follows;

$$HM(G) = \sum_{uv \in E(G)} (\, d_u + d_v)^2.$$

Analogous to this terminology, we introduce the following notions for signed graphs.

Definition 2.1. The $positive\ hyper\ Zagreb\ index$ of a signed graph S, denoted by $HM^+(S)$, is defined as,

$$HM^+(S) = \sum_{uv \in E(G)} (\, d_u^+ + d_v^+)^2,$$

and the $negative\ Zagreb\ index$ of a signed graph S, denoted by $HM^-(S)$, is defined as,

$$HM^-(S) = \sum_{uv \in E(G)} (\, d_u^- + d_v^-)^2.$$

In view of definition 2.1, we have the following theorem.

Theorem 2.2. If S is a signed graph of a graph G, then,

$$HM(G) = HM^+(S) + HM^-(S) + 2M_1^*(S) + 2M_2^*(S).$$

Proof.

$$\begin{aligned} HM(G) &= \sum_{uv \in E(G)} (\, d_u + d_v)^2 \\ &= \sum_{uv \in E(G)} [\,(d_u^+ + d_u^-) + (d_v^+ + d_v^-)]^2. \\ &= \sum_{uv \in E(G)} [\,(d_u^+ + d_v^+)^2 + 2(d_u^+ + d_v^+)(d_u^- + d_v^-) \quad + (d_u^- + d_v^-)^2] \end{aligned}$$

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$$= \sum_{uv\in E(G)} (d_u^+ + d_v^+)^2 + \sum_{uv\in E(G)} (d_u^- + d_v^-)^2 + 2\sum_{uv\in E(G)} [(d_u^+ + d_v^+)(d_u^- + d_v^-)]$$
$$= \sum_{uv\in E(G)} (d_u^+ + d_v^+)^2 + \sum_{uv\in E(G)} (d_u^- + d_v^-)^2 + 2\sum_{uv\in E(G)} (d_u^+ d_u^- + d_v^+ d_v^-)$$
$$+ 2\sum_{uv\in E(G)} (d_u^+ d_v^- + d_v^+ d_u^-)$$
$$= HM^+(S) + HM^-(S) + 2M_1^*(S) + 2M_2^*(S).$$

□

B. Forgotten Topological Indices of Signed Graphs

The *forgotten topological index* of a graph [6] is defined by

$$F(G) = \sum_{uv\in E(G)} d_u^2 + d_v^2.$$

Analogous to this terminology, we introduce the following notions for a signed graph.

Definition 2.3. The *positive forgotten topological index* of a signed graph S is denoted as F^+, is defined to be,

$$F^+(S) = \sum_{uv\in E(G)} (d_u^+)^2 + (d_v^+)^2$$

and the *negative forgotten topological index* of a signed graph S is denoted as F^-, is defined to be,

$$F^-(S) = \sum_{uv\in E(G)} (d_u^-)^2 + (d_v^-)^2.$$

Invoking Definition 2.3, we have,

Theorem 2.4. If S is a signed graph and G is its underlying graph then,

$$F(G) = F^+(S) + F^-(S) + 2M_1^*(S).$$

Proof.

$$F(G) = \sum_{uv\in E(G)} [d_u^2 + d_v^2]$$
$$= \sum_{uv\in E(G)} [(d_u^+ + d_u^-)^2 + (d_v^+ + d_v^-)^2]$$
$$= \sum_{uv\in E(G)} [(d_u^+)^2 + 2d_u^+ d_u^- + (d_u^-)^2 + (d_v^+)^2 + 2d_v^+ d_v^- + (d_v^-)^2]$$

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$$= \sum_{uv\in E(G)} [(d_u^+)^2 + (d_v^+)^2] + \sum_{uv\in E(G)} [(d_u^-)^2 + (d_v^-)^2] + 2\sum_{uv\in E(G)} [d_u^+ d_u^- + d_v^+ d_v^-]$$

$$= F^+(S) + F^-(S) + 2M_1^*(S).$$

□

C. Reduced Second Zagreb Index of Signed Graphs

The *reduced second Zagreb index* of a graph G is defined in [7] as,

$$RM_2(G) = \sum_{uv\in E(G)} (d_u - 1)(d_v - 1).$$

Hence, we introduce the following notions for signed graph as follows.

Definition 2.5. The *positive reduced second Zagreb index* of a signed graph S is denoted as $RM_2^+(S)$, is defined to be,

$$RM_2^+(S) = \sum_{uv\in E(G)} (d_u^+ - 1)(d_v^+ - 1)$$

and the *negative reduced second Zagreb index* of a signed graph S is denoted as $RM_2^-(S)$, is defined to be,

$$RM_2^-(S) = \sum_{uv\in E(G)} (d_u^- - 1)(d_v^- - 1).$$

Theorem 2.6. If S is a signed graph and G is its underlying graph then,

$$RM_2(G) = RM_2^+(S) + RM_2^-(S) + M_2^*(S) - |E(G)|.$$

Proof.

$$RM_2(G) = \sum_{uv\in E(G)} (d_u - 1)(d_v - 1)$$

$$= \sum_{uv\in E(G)} (d_u^+ + d_u^- - 1)(d_v^+ + d_v^- - 1)$$

$$= \sum_{uv\in E(G)} \{d_u^+ (d_v^+ + d_v^- - 1) + d_u^-(d_v^+ + d_v^- - 1) \quad - 1(d_v^+ + d_v^- - 1)\}$$

$$= \sum_{uv\in E(G)} \{(d_u^+ d_v^+ + d_u^+ d_v^- - d_u^+) + (d_u^- d_v^+ + d_u^- d_v^- - d_u^-) - (d_v^+ + d_v^- - 1)\}$$

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$$= \sum_{uv\in E(G)} \{(d_u^+ d_v^+ - d_u^+ - d_v^+ + 1) + (d_u^- d_v^- - d_u^- - d_v^- + 1) + (d_u^+ d_v^- + d_u^- d_v^- - 1)\}$$

$$= \sum_{uv\in E(G)} (d_u^+ - 1)(d_v^+ - 1) + \sum_{uv\in E(G)} (d_u^- - 1)(d_v^- - 1)$$

$$+ \sum_{uv\in E(G)} (d_u^+ d_v^- + d_u^- d_v^+) - \sum_{uv\in E(G)} 1$$

$$= RM_2^+(S) + RM_2^-(S) + M_2^*(S) - |E(G)|.$$

□

D. First and Second Zagreb Coindices

The first and second Zagreb coindices contributes to the non-adjacent pair of vertices when computing topological indices. Here, we take the sum that runs over all the edges that are in the complement of graph G. Such quantities are called Zagreb coindices. This new pair of invariants were introduced in [5].

Formally, the $first\ Zagreb\ coindex$ of a graph G is defined in [5] as,

$$\overline{M}_1(G) = \sum_{uv\notin E(G)} d_u + d_v,$$

and the $second\ Zagreb\ coindex$ of a graph G is defined in [5] as,

$$\overline{M}_2(G) = \sum_{uv\notin E(G)} d_u\, d_v.$$

The reader may note that the Zagreb coindices of G are different from Zagreb indices of G; the defining sum runs over $E(\overline{G})$, but degrees are with respect to graph G (see [1]).

Analogous to these definitions, we introduce the following notions for signed graph as follows.

Definition 2.7. The $positive\ first\ Zagreb\ coindex$ of a graph G denoted as $\overline{M}_1^+(S)$ is defined as,

$$\overline{M}_1^+(S) = \sum_{uv\notin E(G)} d_u^+ + d_v^+,$$

and the $negative\ first\ Zagreb\ coindex$ of a graph G denoted as $\overline{M}_1^-(S)$ is defined as,

$$\overline{M}_1^-(S) = \sum_{uv\notin E(G)} d_u^- + d_v^-.$$

Definition 2.8. The $positive\ second\ Zagreb\ coindex$ of a graph G denoted as $\overline{M}_2^+(S)$ is defined as,

$$\overline{M}_2^+(S) = \sum_{uv \notin E(G)} d_u^+ \, d_v^+,$$

and the $negative\ second\ Zagreb\ coindex$ of a graph G denoted as $\overline{M}_2^-(S)$ is defined as,

$$\overline{M}_2^-(S) = \sum_{uv \notin E(G)} d_u^- \, d_v^-.$$

The following theorem is obtained in the view of Definition 2.7 and Definition 2.8.

Theorem 2.9. If S is a signed graph and G is its underlying graph, then

$$\overline{M}_1(G) = \overline{M}_1^+(S) + \overline{M}_1^-(S).$$

Proof.

$$\begin{aligned}
\overline{M}_1(G) &= \sum_{uv \notin E(G)} d_u + d_v \\
&= \sum_{uv \notin E(G)} (\, d_u^+ + d_u^-) + (d_v^+ + d_v^-) \\
&= \sum_{uv \notin E(G)} (\, d_u^+ + d_v^+) + (d_u^- + d_v^-) \\
&= \sum_{uv \notin E(G)} (\, d_u^+ + d_v^+) + \sum_{uv \notin E(G)} (\, d_u^- + d_v^-) \\
&= \overline{M}_1^+(S) + \overline{M}_1^-(S).
\end{aligned}$$

□

Theorem 2.10. If S is a signed graph and G its underlying graph, then

$$\overline{M}_2(G) = \overline{M}_2^+(S) + \overline{M}_2^-(S) + \sum_{uv \notin E(G)} (\, d_u^+ d_v^- + d_u^- d_v^+)$$

Proof.

$$\begin{aligned}
\overline{M}_2(G) &= \sum_{uv \notin E(G)} d_u \, d_v \\
&= \sum_{uv \notin E(G)} (\, d_u^+ + d_u^-)(d_v^+ + d_v^-) \\
&= \sum_{uv \notin E(G)} (\, d_u^+ d_v^+ + d_u^+ d_v^- + d_u^- d_v^+ + d_u^- d_v^-)
\end{aligned}$$

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$$= \sum_{uv \notin E(G)} (d_u^+ d_v^+) + \sum_{uv \notin E(G)} (d_u^- d_v^-) + \sum_{uv \notin E(G)} (d_u^+ d_v^- + d_u^- d_v^+)$$

$$= \overline{M}_2^+(S) + \overline{M}_2^-(S) + \sum_{uv \notin E(G)} (d_u^+ d_v^- + d_u^- d_v^+)$$

□

II. TOPOLOGICAL INDICES BASED ON THE NET DEGREE OF VERTICES

The *net degree* of a signed graph S, denoted by $d_S^{\pm}$, is defined in [11] as $d_S^{\pm} = d_S^+ - d_S^-$. The signed graph S is said to be *net regular* if every vertex has the same net degree. We use the notation $\hat{d}_S(v)$ as denoted in [18] instead of the above notation to represent the net degree of a vertex in a signed graph S.

The *first Zagreb net index* for a signed graph S is denoted by $M_1(S)$ is defined as,

$$M_1(S) = \sum_{u \in V(S)} \hat{d}_u^2$$

and the *second Zagreb net index* of S is denoted by $M_2(S)$ is defined as,

$$M_2(S) = \sum_{uv \in E(S)} \widehat{d_u}\, \widehat{d_v}.$$

E. Hyper Zagreb Net Index

Definition 3.1. Let S be a signed graph and let $\widehat{d_i}$ denote the net degree of a vertex in the signed graph S. The *hyper Zagreb net index* of the signed graph S is denoted by $HM(S)$ and is defined as,

$$HM(S) = \sum_{uv \in E(S)} (\widehat{d_i^2} + \widehat{d_j^2})$$

In view of Definition 3.1 in this section, we have the following theorem.

Theorem 3.2. If S is a signed graph of a graph G, then,

$$HM(S) = HM^+(S) + HM^-(S) - 2M_1^*(S) - 2M_2^*(S).$$

Proof.

$$HM(G) = \sum_{uv \in E(S)} (\hat{d}_u + \hat{d}_v)^2$$

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$$= \sum_{uv\in E(S)} [(d_u^+ - d_u^-) + (d_v^+ - d_v^-)]^2$$
$$= \sum_{uv\in E(S)} [(d_u^+ + d_v^+) - (d_u^- + d_v^-)]^2$$
$$= \sum_{uv\in E(S)} [(d_u^+ + d_v^+)^2 - 2(d_u^+ + d_v^+)(d_u^- + d_v^-) + (d_u^- + d_v^-)^2]$$
$$= \sum_{uv\in E(S)} (d_u^+ + d_v^+)^2 + \sum_{uv\in E(S)} (d_u^- + d_v^-)^2 - 2\sum_{uv\in E(S)} [(d_u^+ + d_v^+)(d_u^- + d_v^-)]$$
$$= \sum_{uv\in E(S)} (d_u^+ + d_v^+)^2 + \sum_{uv\in E(S)} (d_u^- + d_v^-)^2 - 2\sum_{uv\in E(S)} (d_u^+ d_u^- + d_v^+ d_v^-)$$
$$-2\sum_{uv\in E(S)} (d_u^+ d_v^- + d_v^+ d_u^-)$$
$$= HM^+(S) + HM^-(S) - 2M_1^*(S) - 2M_2^*(S).$$

□

F. Forgotten Topological Net Index

Definition 3.3. The *forgotten topological net index* of a signed graph S, denoted by $F(S)$, is defined as,

$$F(S) = \sum_{uv\in E(S)} \hat{d}_u^2 + \hat{d}_v^2.$$

From the Definition 3.3, we have the following theorem.

Theorem 3.4. For a signed graph S,

$$F(S) = F^+(S) + F^-(S) - 2M_1^*(S).$$

Proof.

$$F(S) = \sum_{uv\in E(S)} \hat{d}_u^2 + \hat{d}_v^2$$
$$= \sum_{uv\in E(S)} [(d_u^+ - d_u^-)^2 + (d_v^+ - d_v^-)^2]$$
$$= \sum_{uv\in E(S)} [(d_u^+)^2 - 2d_u^+ d_u^- + (d_u^-)^2 + (d_v^+)^2 - 2d_v^+ d_v^- + (d_v^-)^2]$$
$$= \sum_{uv\in E(S)} [(d_u^+)^2 + (d_v^+)^2] + \sum_{uv\in E(S)} [(d_u^-)^2 + (d_v^-)^2]$$
$$- 2\sum_{uv\in E(S)} [d_u^+ d_u^- + d_v^+ d_v^-]$$

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$= F^+(S) + F^-(S) - 2M_1^*.$

□

G. Reduced Second Zagreb Net Index

Definition 3.5. The *reduced second Zagreb net index* of a signed graph S is defined by,

$$RM_2(S) = \sum_{uv\in E(S)} (\hat{d}_u - 1)(\hat{d}_v - 1).$$

Invoking definition 3.5 we have the following the theorem.

Theorem 3.6. If S is a signed graph, then

$$RM_2(S) = RM_2^+(S) - RM_2^-(S) - M_2^*(S) - |E(S)|.$$

Proof.

$$\begin{aligned}
RM_2(S) &= \sum_{uv\in E(S)} (\hat{d}_u - 1)(\hat{d}_v - 1)\\
&= \sum_{uv\in E(S)} (d_u^+ - d_u^- - 1)(d_v^+ - d_v^- - 1)\\
&= \sum_{uv\in E(S)} \{d_u^+ (d_v^+ - d_v^- - 1) - d_u^-(d_v^+ - d_v^- - 1) - 1(d_v^- + d_v^- - 1)\}\\
&= \sum_{uv\in E(S)} \{(d_u^+ d_v^+ - d_u^+ d_v^- - d_u^+ \ )\\
&\quad -(d_u^- d_v^+ + d_u^- d_v^- \ - d_u^-) - (d_v^+ + d_v^- - 1)\}\\
&= \sum_{uv\in E(S)} \{(d_u^+ d_v^+ - d_u^+ - d_v^+ + 1) - (d_u^- d_v^- - d_u^- - d_v^- + 1) - (d_u^+ d_v^- + d_u^- d_v^-\\
&\quad + 1)\}\\
&= \sum_{uv\in E(S)} (d_u^+ - 1)(d_v^+ - 1) - \sum_{uv\in E(S)} (d_u^- - 1)(d_v^- - 1) - \sum_{uv\in E(S)} (d_u^+ d_v^- + d_u^- d_v^+)\\
&\quad - \sum_{uv\in E(S)} 1\\
&= RM_2^+(S) - RM_2^-(S) - M_2^*(S) - |E(S)|.
\end{aligned}$$

□

H. First and Second Zagreb Net Coindices

Definition 3.7. The *first Zagreb net coindex* of a graph G is defined [5] as,

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$$\overline{M}_1(S) = \sum_{uv \notin E(G)} \hat{d}_u + \hat{d}_v,$$

and the *second Zagreb net coindex* of a graph G is defined [5] as,

$$\overline{M}_2(S) = \sum_{uv \notin E(G)} \hat{d}_u \hat{d}_v.$$

Theorem 3.8. If S is a signed graph and G is its underlying graph then,

$$\overline{M}_1(S) = \overline{M}_1^{+}(S) - \overline{M}_1^{-}(S).$$

Proof.

$$\begin{aligned}
\overline{M}_1(S) &= \sum_{uv \notin E(G)} \hat{d}_u + \hat{d}_v \\
&= \sum_{uv \notin E(G)} (d_u^+ - d_u^-) + (d_v^+ - d_v^-) \\
&= \sum_{uv \notin E(G)} (d_u^+ + d_v^+) - (d_u^- + d_v^-) \\
&= \sum_{uv \notin E(G)} (d_u^+ + d_v^+) - \sum_{uv \notin E(G)} (d_u^- + d_v^-) \\
&= \overline{M}_1^{+}(S) - \overline{M}_1^{-}(S).
\end{aligned}$$

□

Theorem 3.9. If S is a signed graph and G is its underlying graph, then

$$\overline{M}_2(S) = \overline{M}_2^{+}(S) + \overline{M}_2^{-}(S) - \sum_{uv \notin E(G)} (d_u^+ d_v^- + d_u^- d_v^+).$$

Proof.

$$\begin{aligned}
\overline{M}_2(S) &= \sum_{uv \notin E(G)} \hat{d}_u \hat{d}_v \\
&= \sum_{uv \notin E(G)} (d_u^+ - d_u^-)(d_v^+ - d_v^-) \\
&= \sum_{uv \notin E(G)} (d_u^+ d_v^+ - d_u^+ d_v^- - d_u^- d_v^+ + d_u^- d_v^-) \\
&= \sum_{uv \notin E(G)} (d_u^+ d_v^+) + \sum_{uv \notin E(G)} (d_u^- d_v^-) - \sum_{uv \notin E(G)} (d_u^+ d_v^- + d_u^- d_v^+) \\
&= \overline{M}_2^{+}(S) + \overline{M}_2^{-}(S) - \sum_{uv \notin E(G)} (d_u^+ d_v^- + d_u^- d_v^+).
\end{aligned}$$

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Conclusion: This work has introduced different versions of degree based topological indices such as hyper Zagreb indices, forgotten topological indices, reduced second Zagreb indices, first and second Zagreb coindices of signed graphs. We here considered the degree of the vertices.

The second part of paper has discussed the topological indices of signed graphs on the basis of net degree of the vertices of it which we call as net index and the relation between these topological indices of signed graph to the underlying graphs is obtained.

There are lots of open problems available in this area such as determining various other degree based topological indices for signed graphs such as bell index, third Zagreb index, the generalized first Zagreb index, Atom-Bond Connectivity indices, Geometric-Arithmetic indices, etc. These topological indices have diverse application potential.

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